Current Distribution Inference from MSE Coherence Imaging using Bayesian Tomography

(Assesment of IMSE capabilities for Tokamaks and Stellartors - ASDEX Upgrade and W7X)

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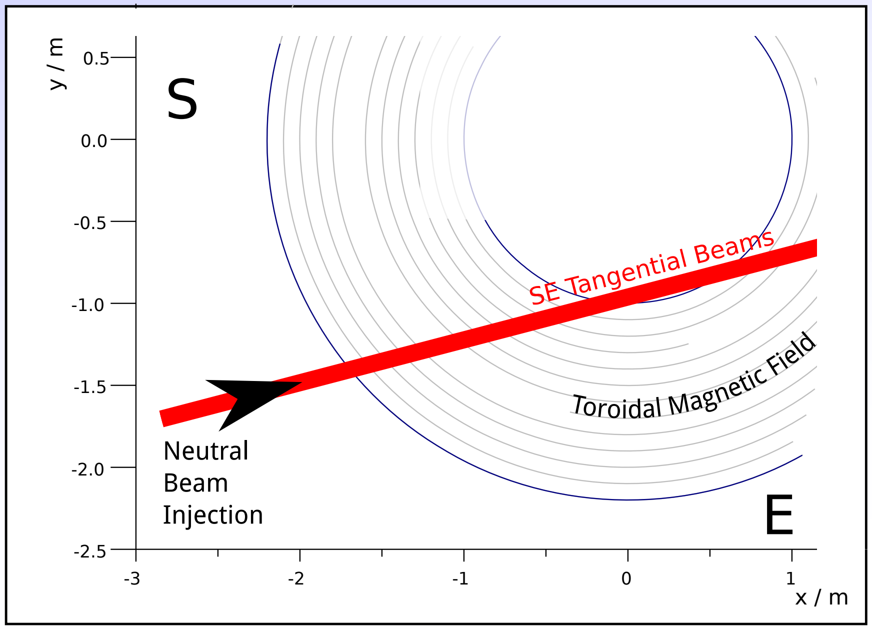
- Imaging MSE.
    - Introduction
    - Forward Model

- ASDEX Upgrade Instrument.
    - 2D measurements under Axisymmetry.
    - Current Tomography.

- W7X Instrument.
    - Capability Analysis
    - Bayesian Inference using Funtion Parameterisation.

Introduction - Motional Stark Effect



Neutral beam atoms injected into plasma.
Excited by plasma, then emit Hα/Dα radiation.

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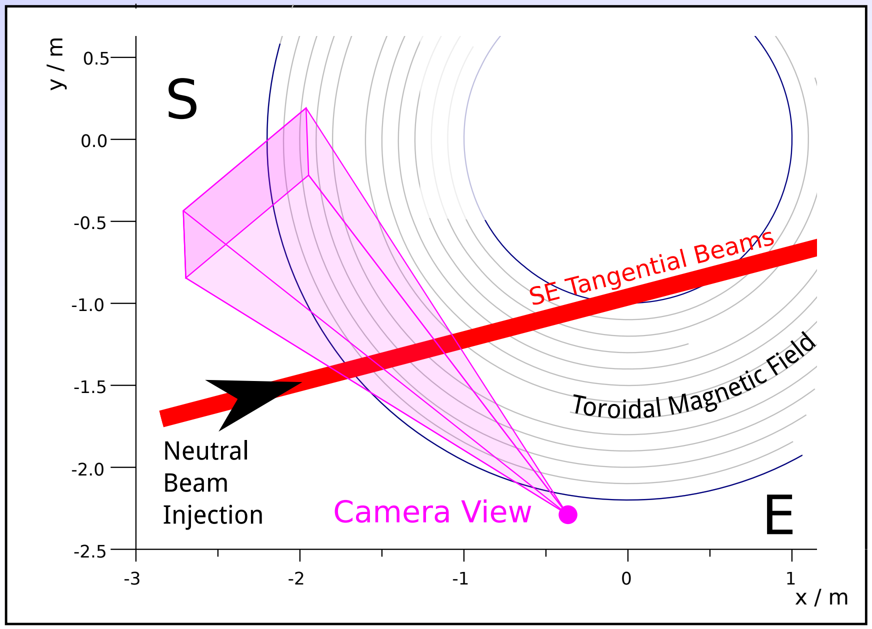

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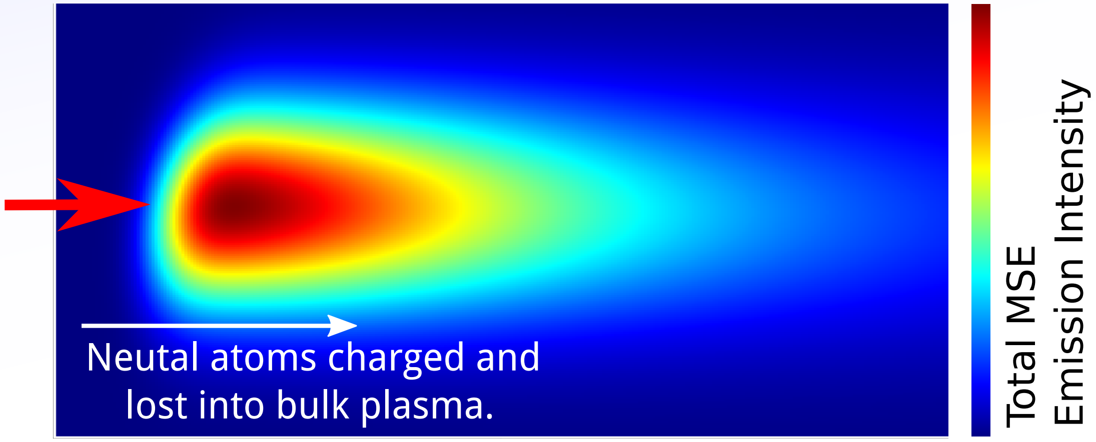

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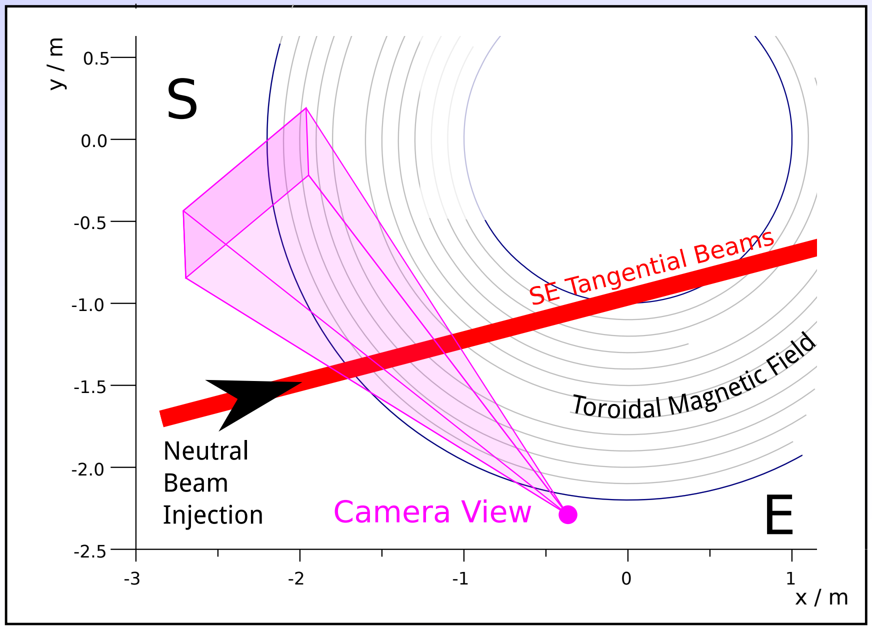

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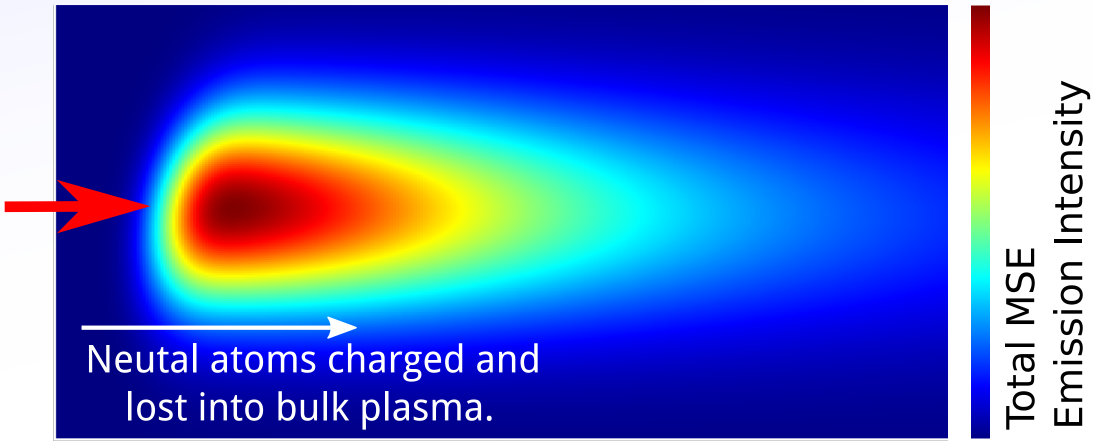
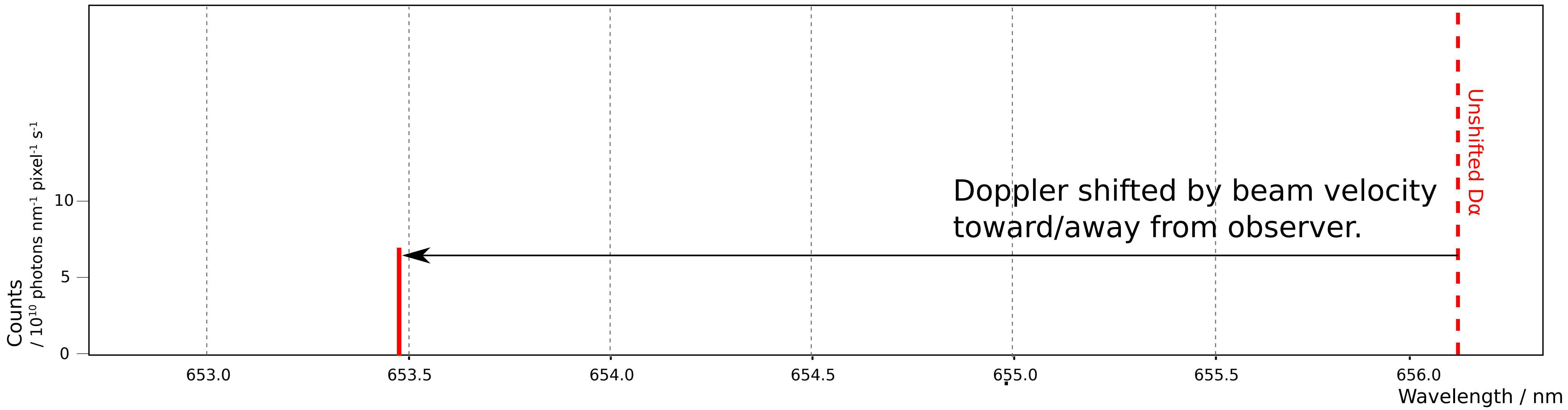


Spectrum from a single pixel:



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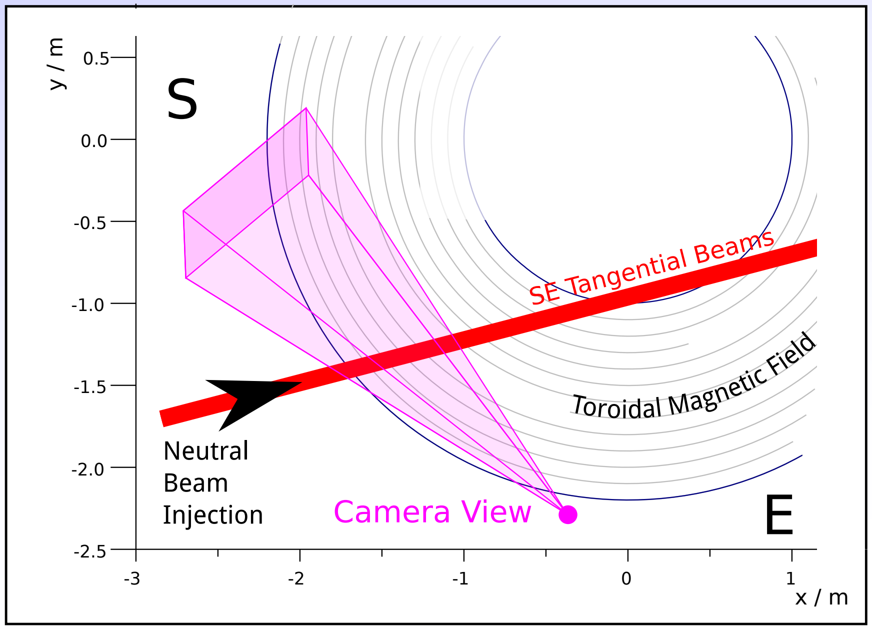

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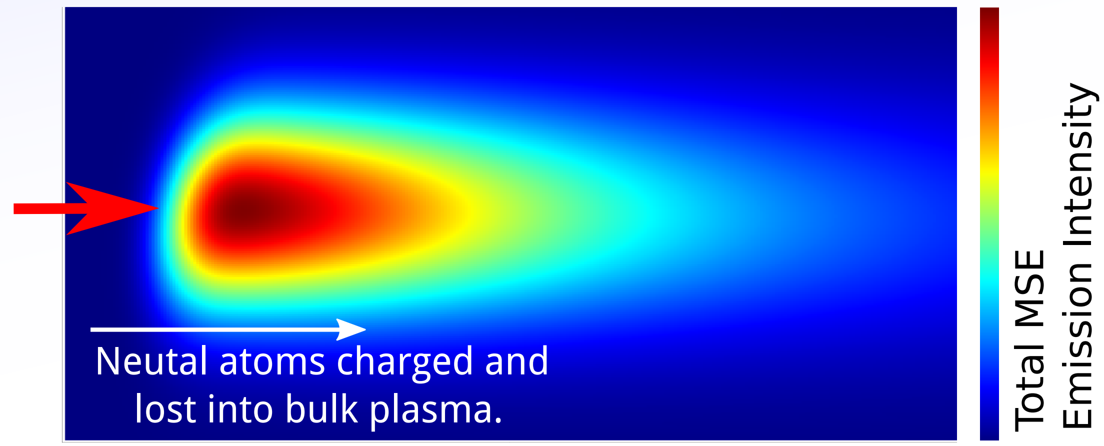

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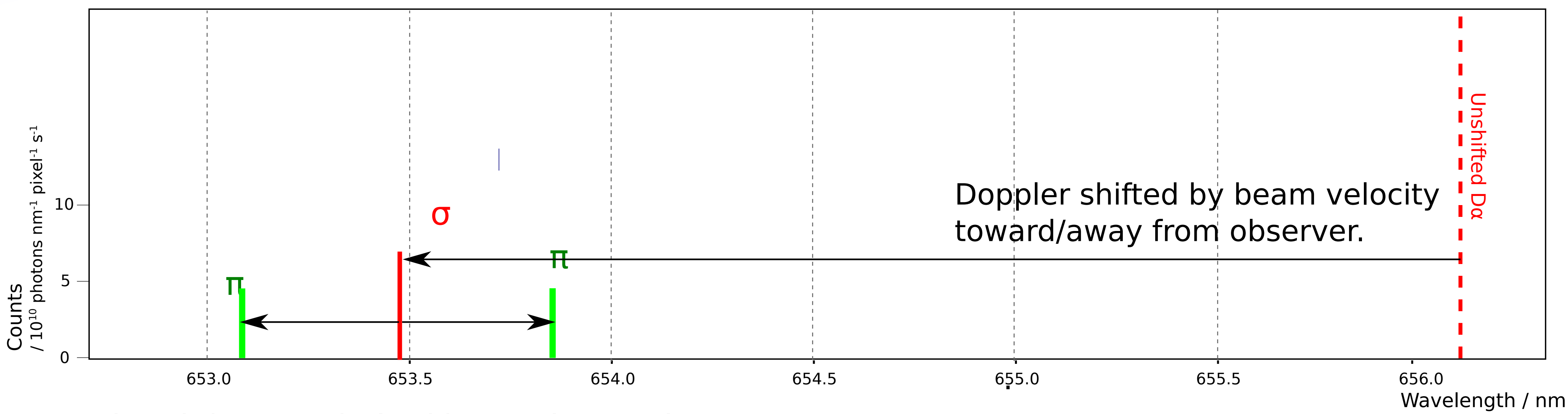

Stark split by electric field in rest frame of atom:

$\boldsymbol{E} = \boldsymbol{v} \times \boldsymbol{B}$

Roughly: π polarised parallel to E.
σ polarised perp' to E.

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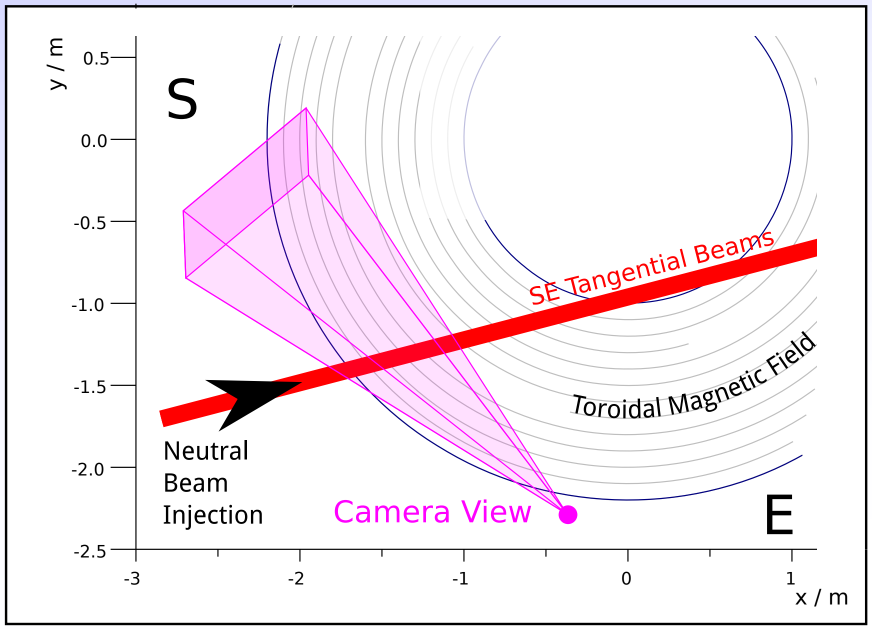

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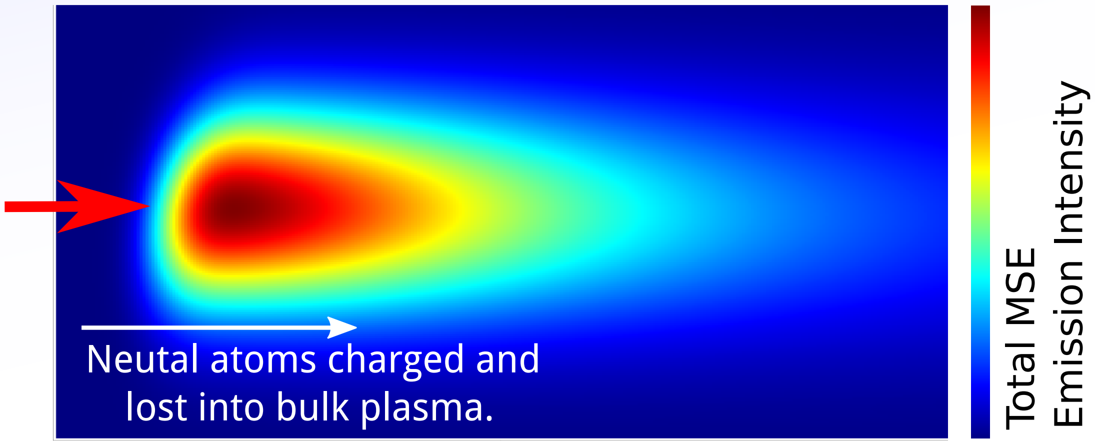

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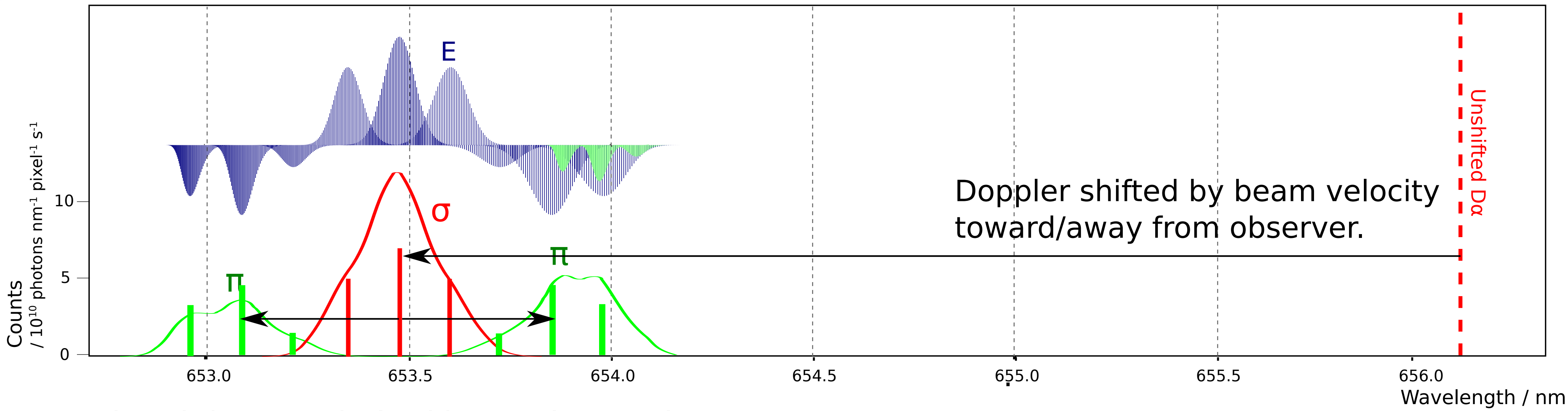

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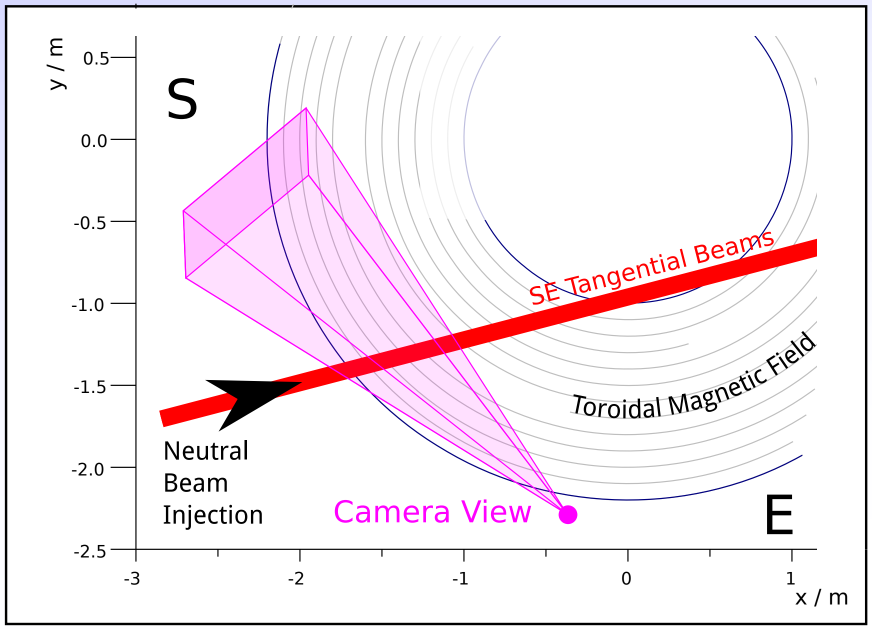

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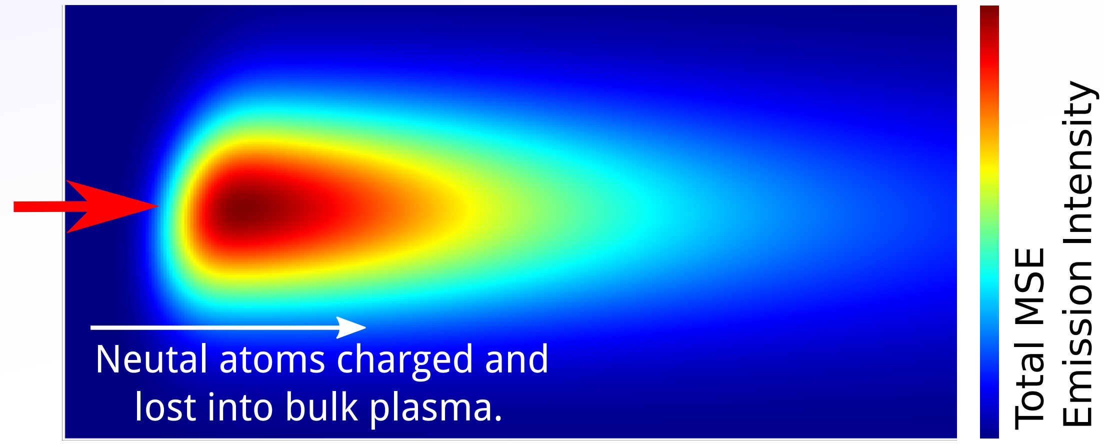

Complications:
Energy components, Doppler broadening,
Beam divergence, Line integration etc.

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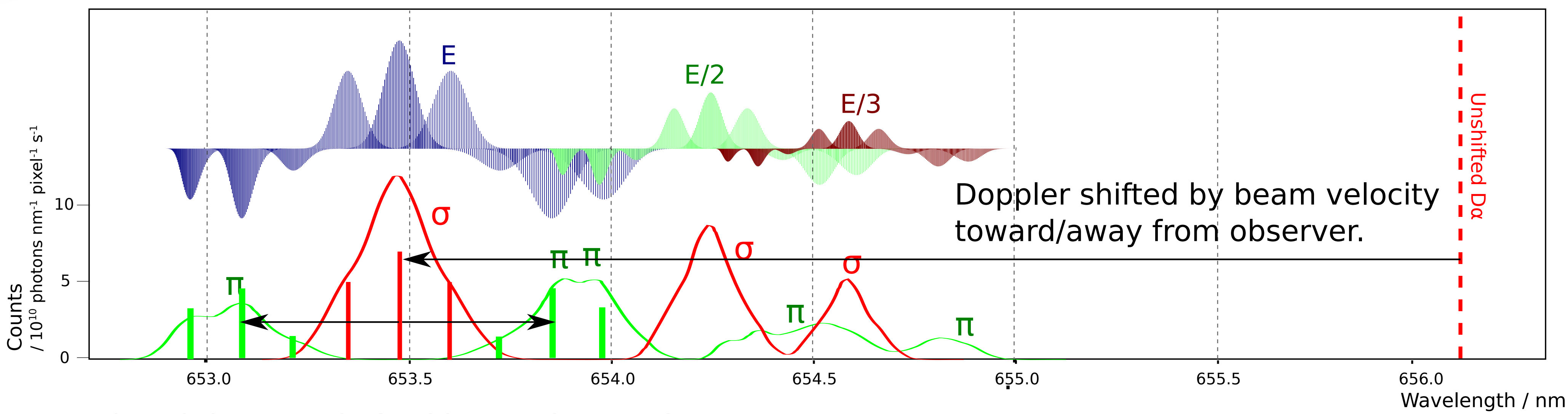

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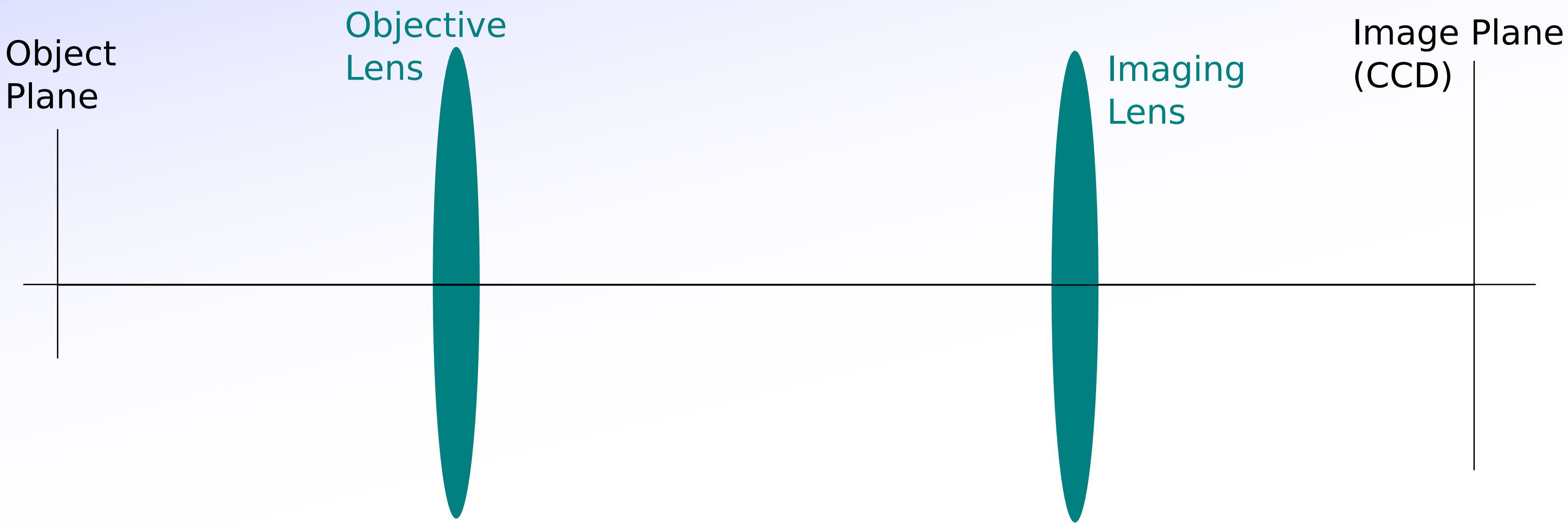
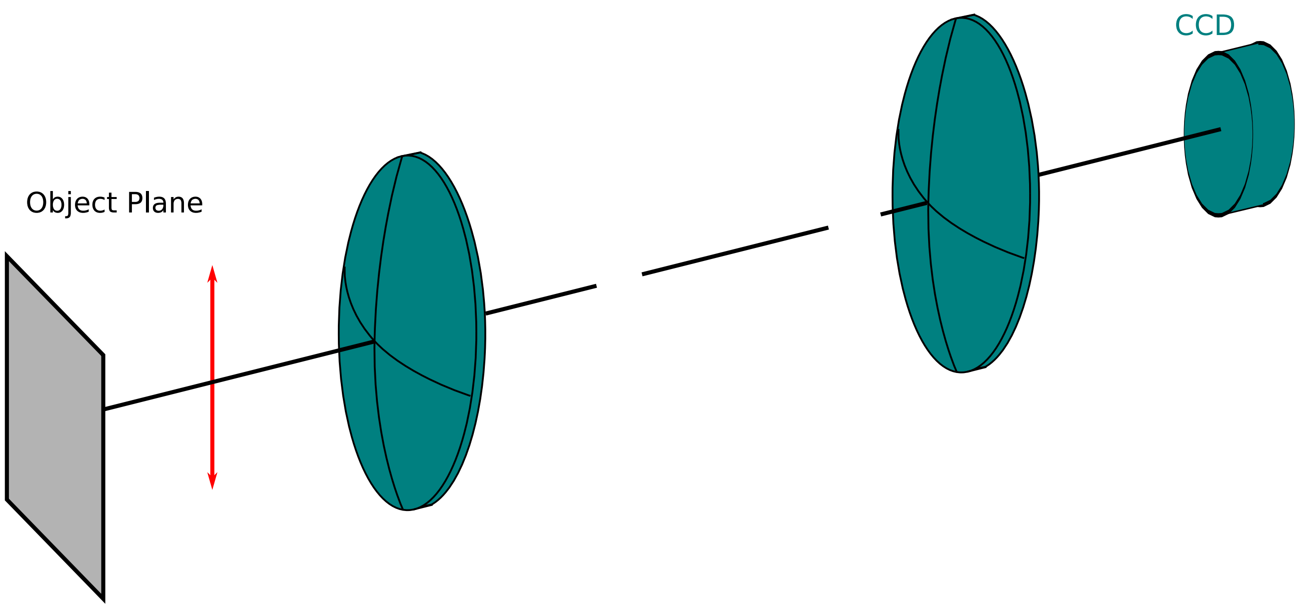
Savart Plates

Savart Plate: Angle dependent phase shift --> Interference pattern accross image.





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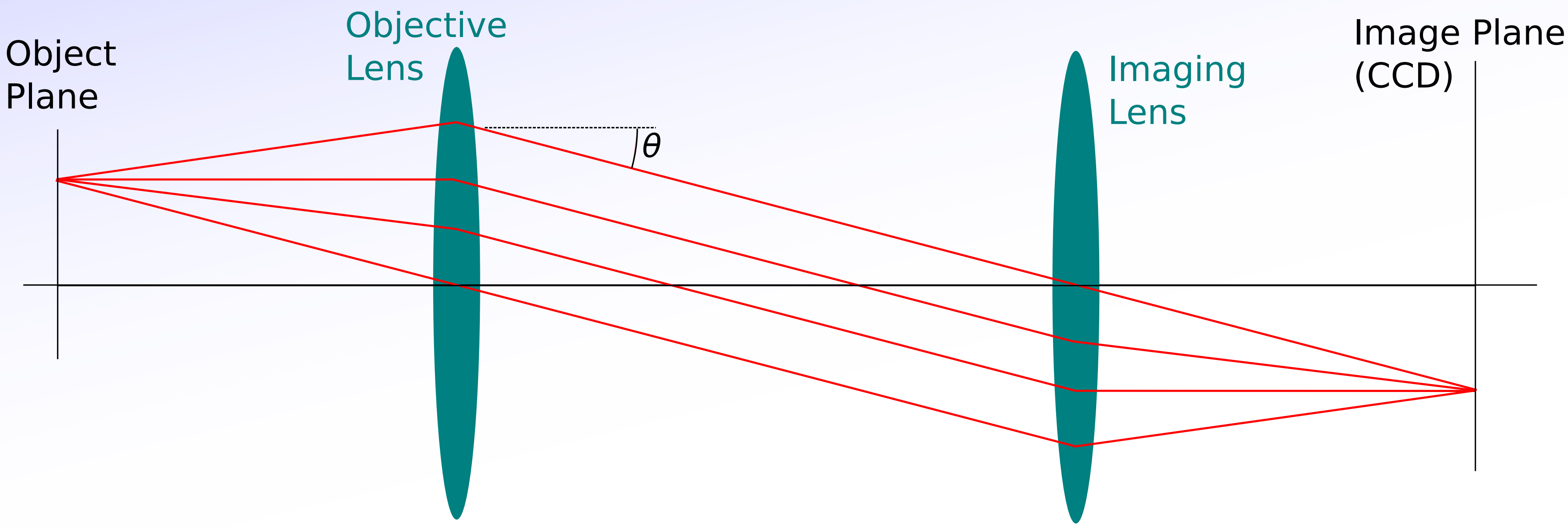

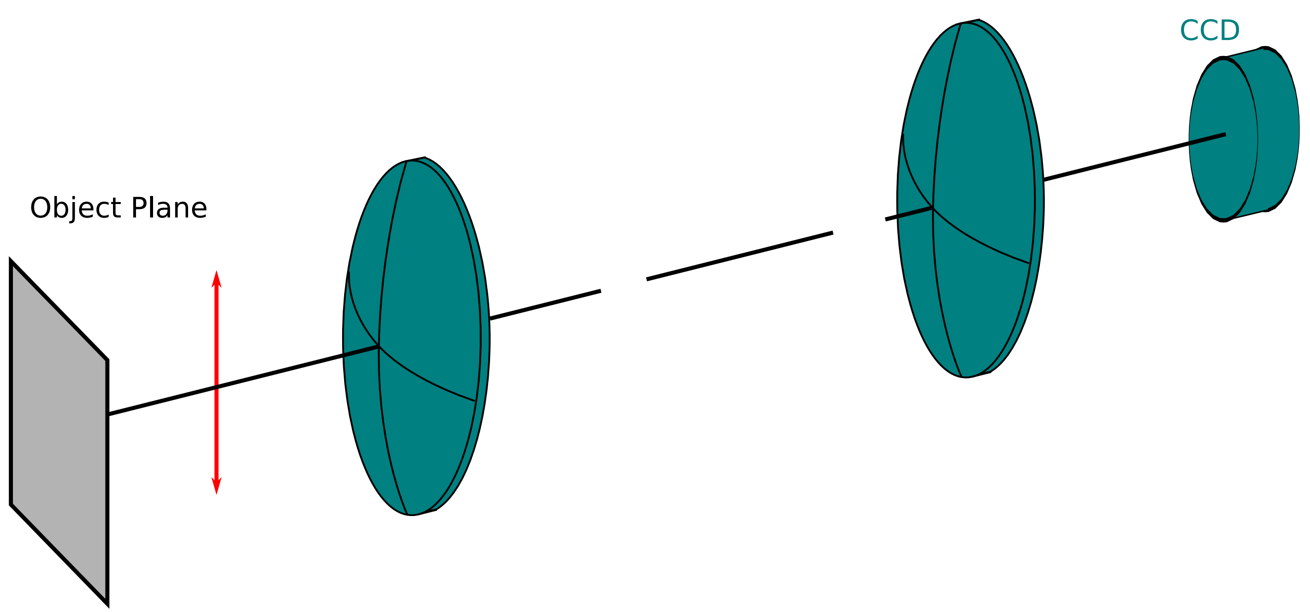

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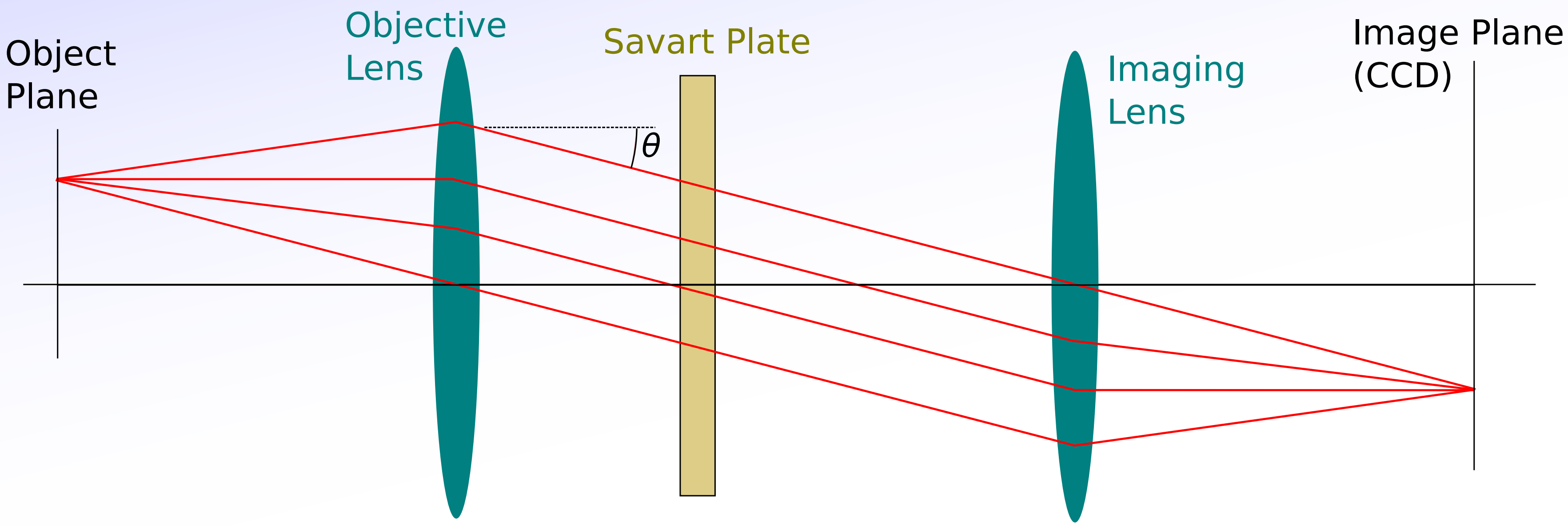

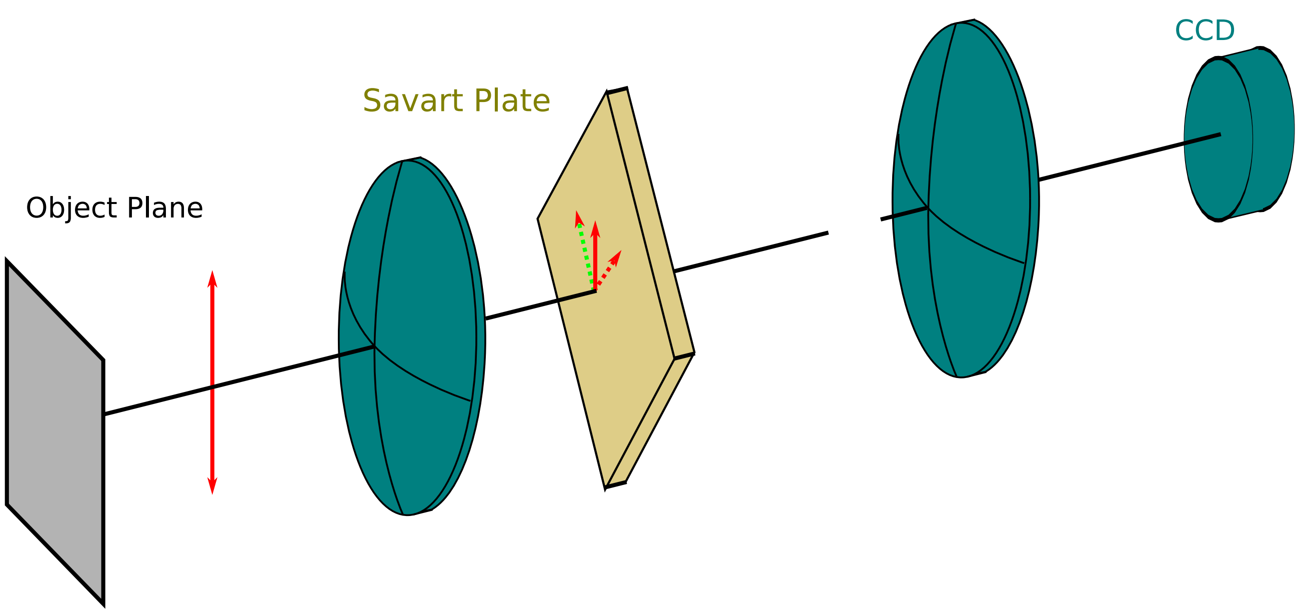

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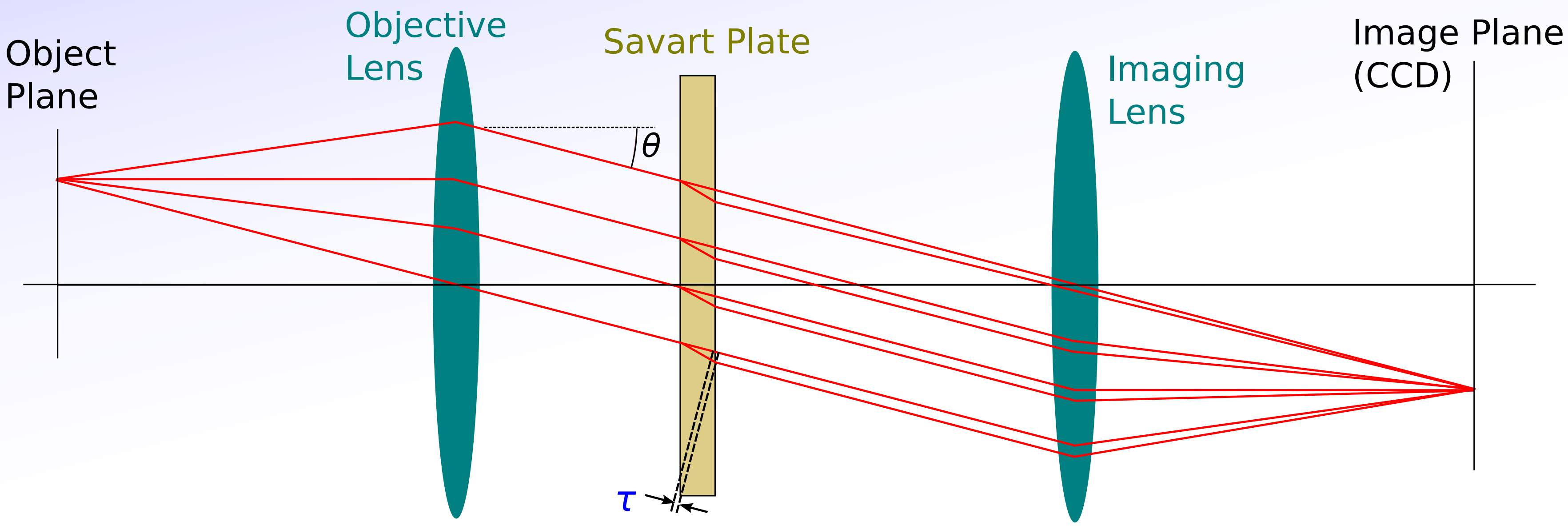

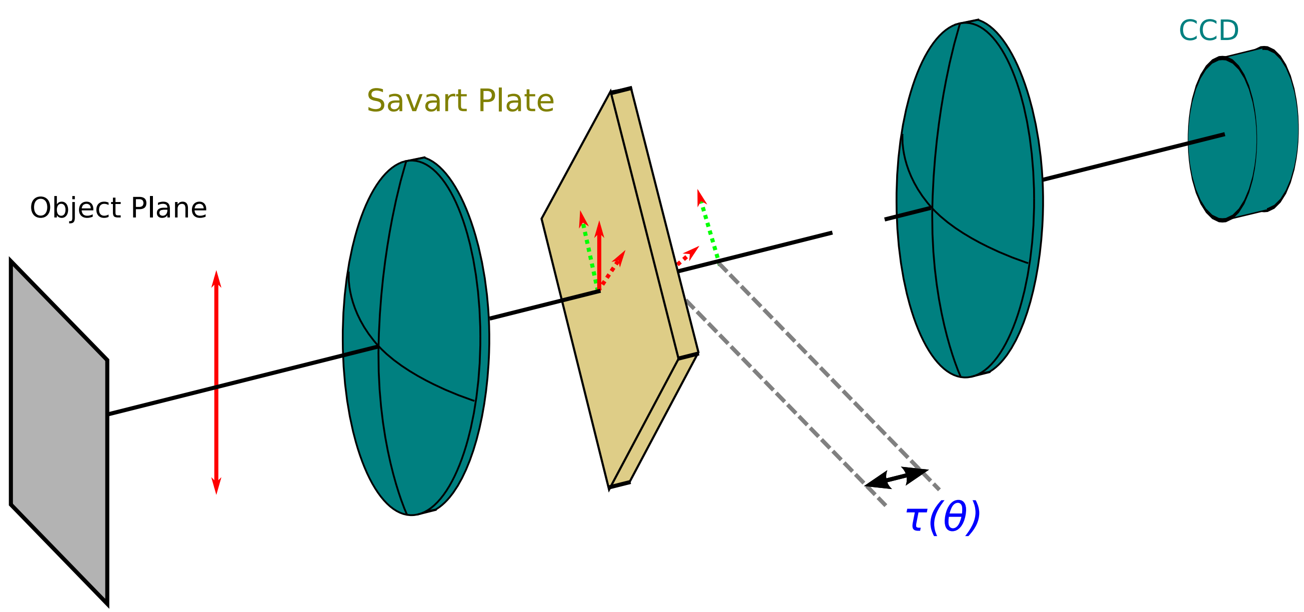

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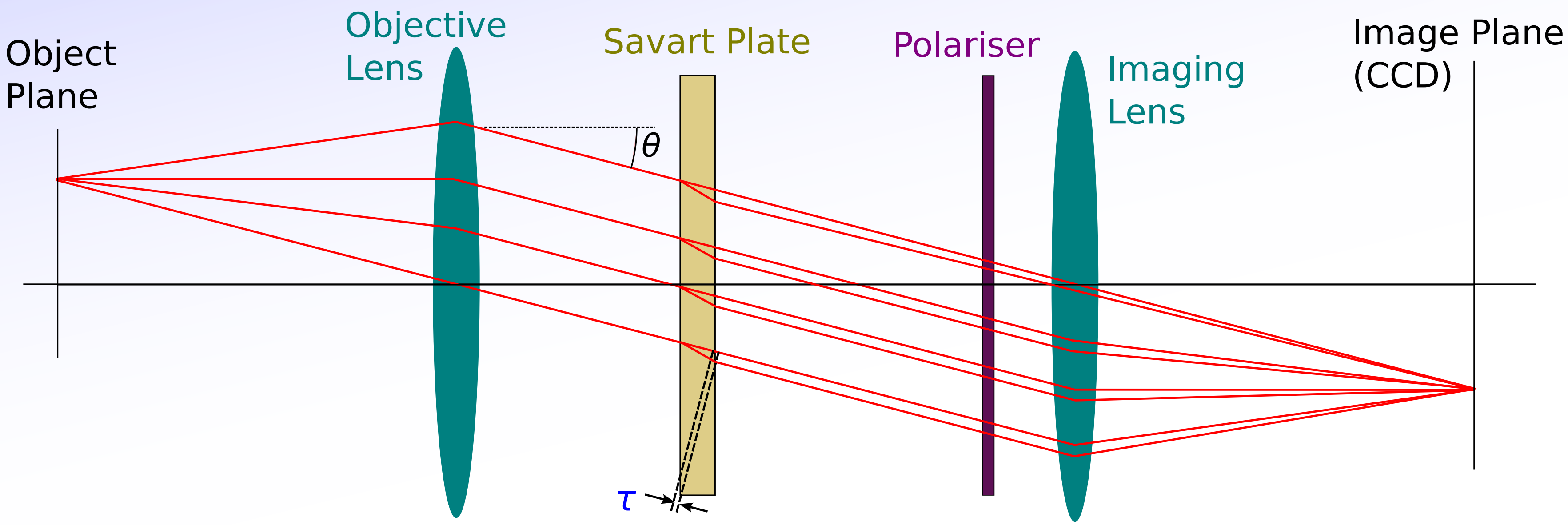

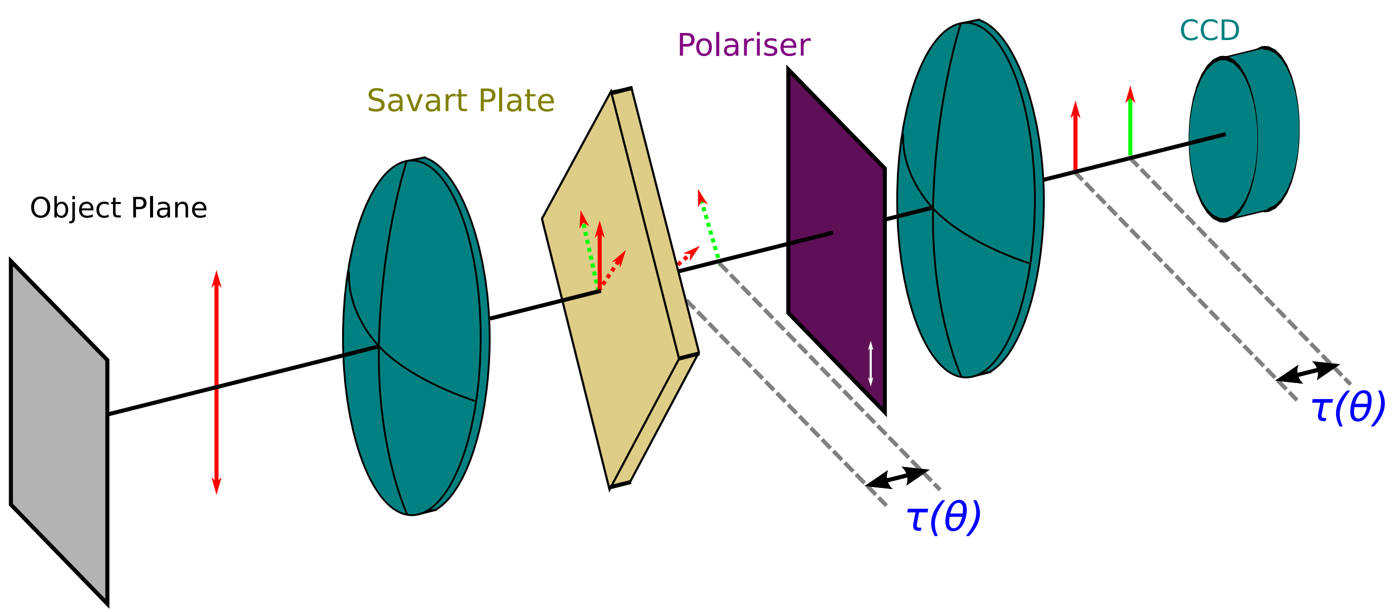

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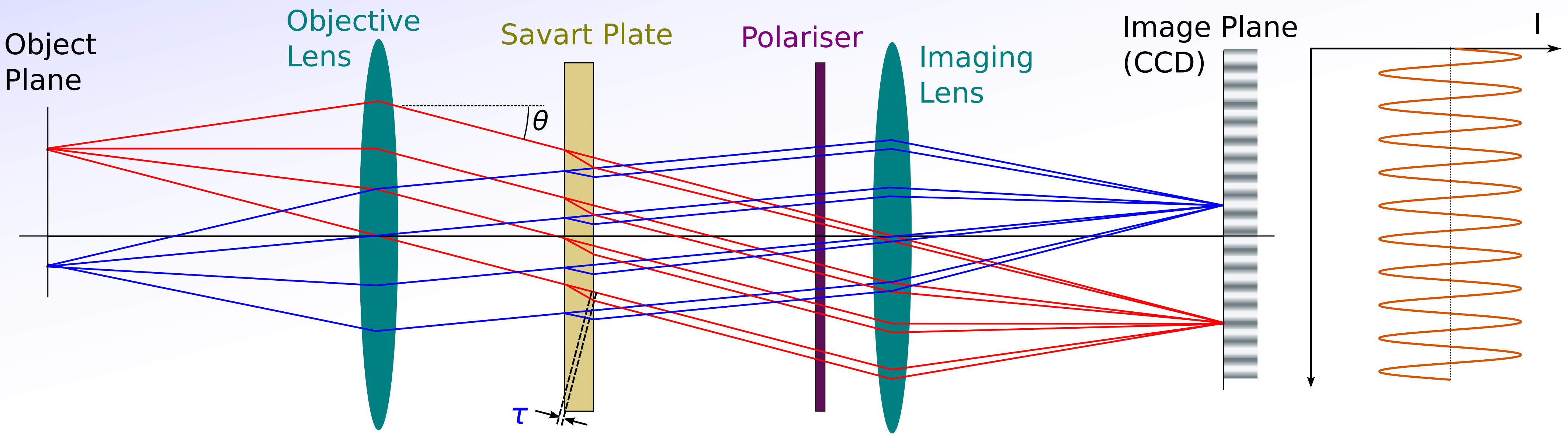

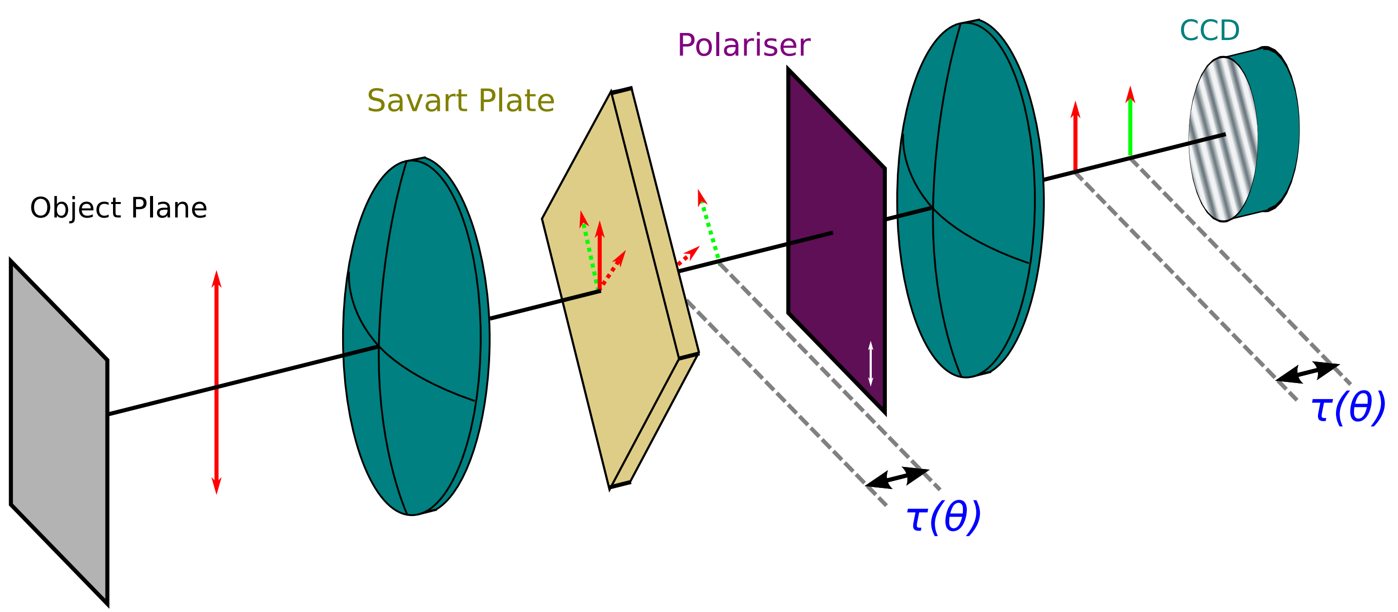

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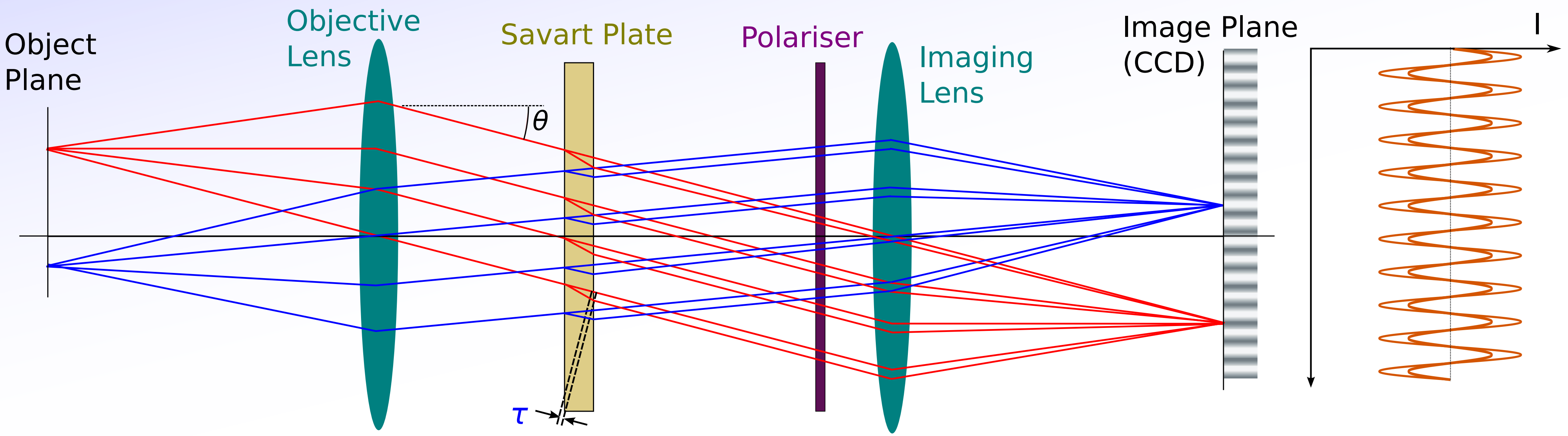
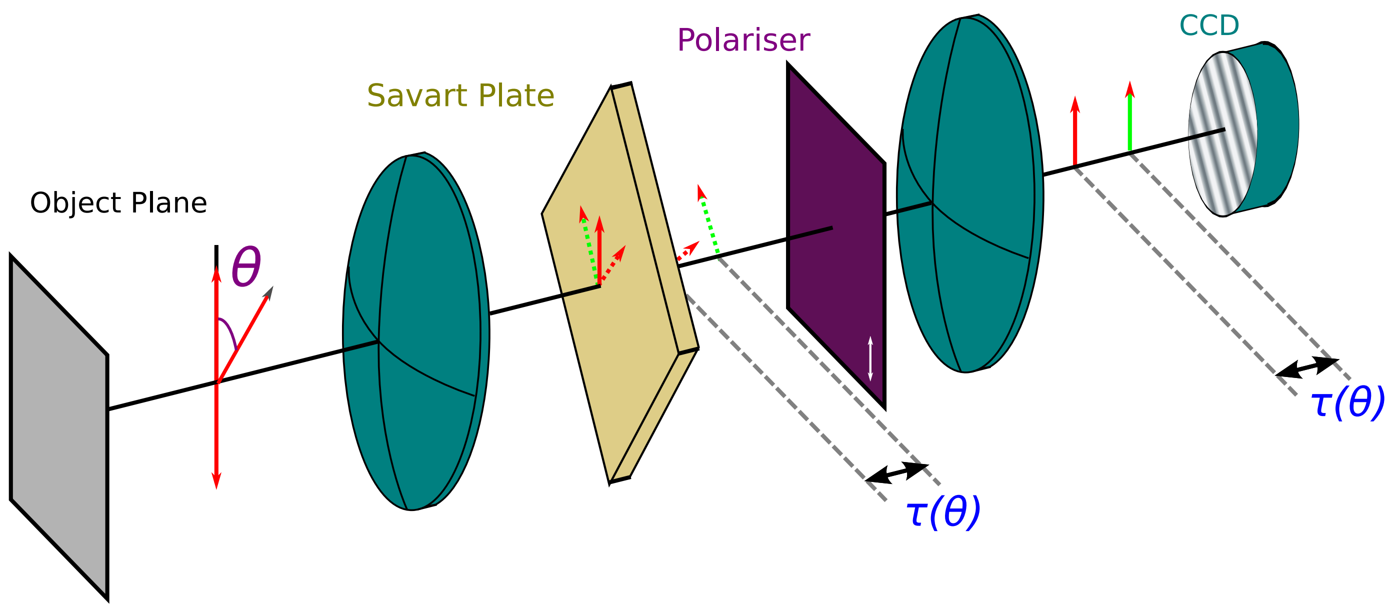


Oscillation amplitude proportional to polarisation angle.

$$I \propto 1 + cos2\theta \, cos(x)$$



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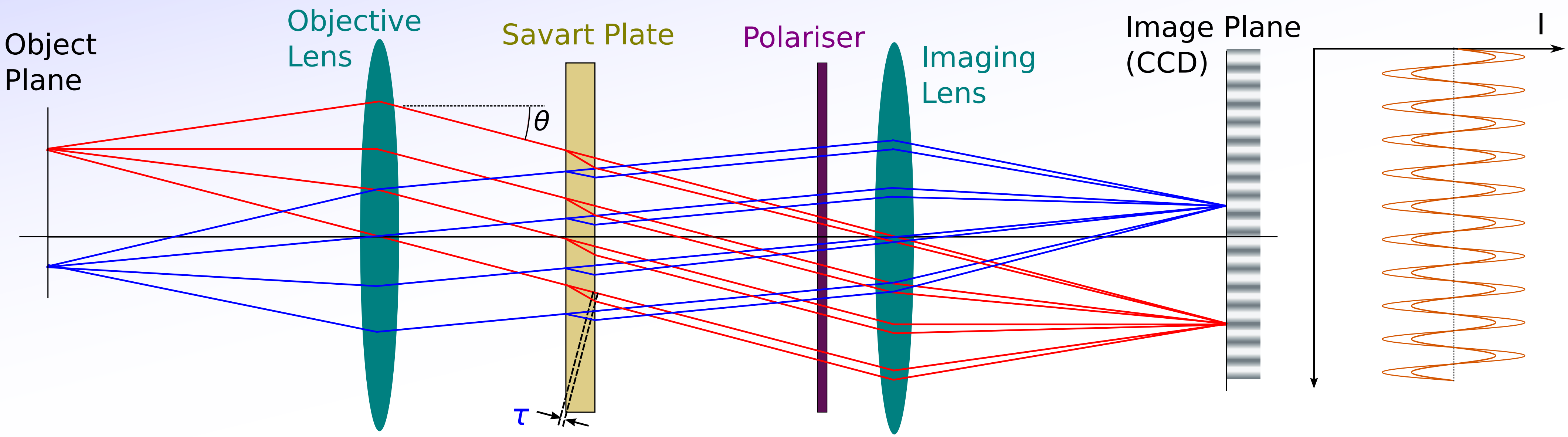
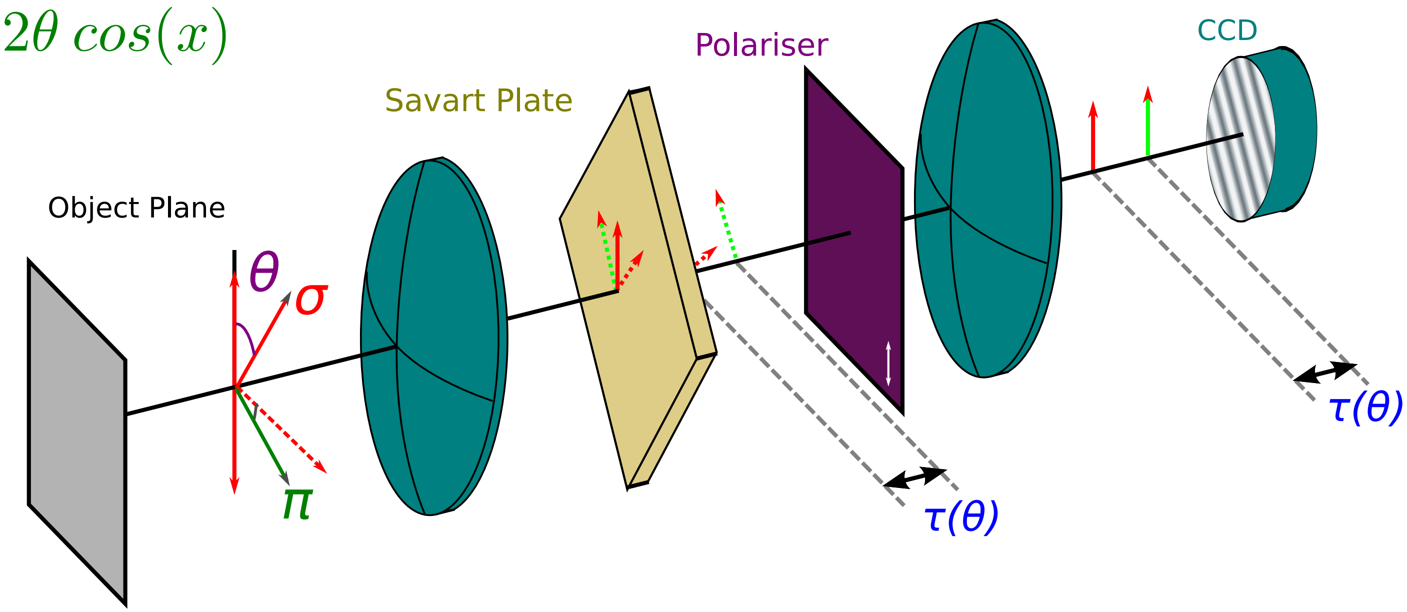


Oscillation amplitude proportional to polarisation angle.

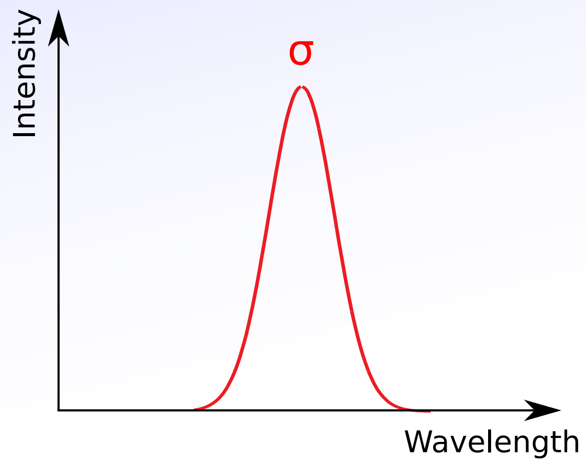
$$I \propto 1 + cos2\theta\ cos(x) - cos2\theta\ cos(x)$$

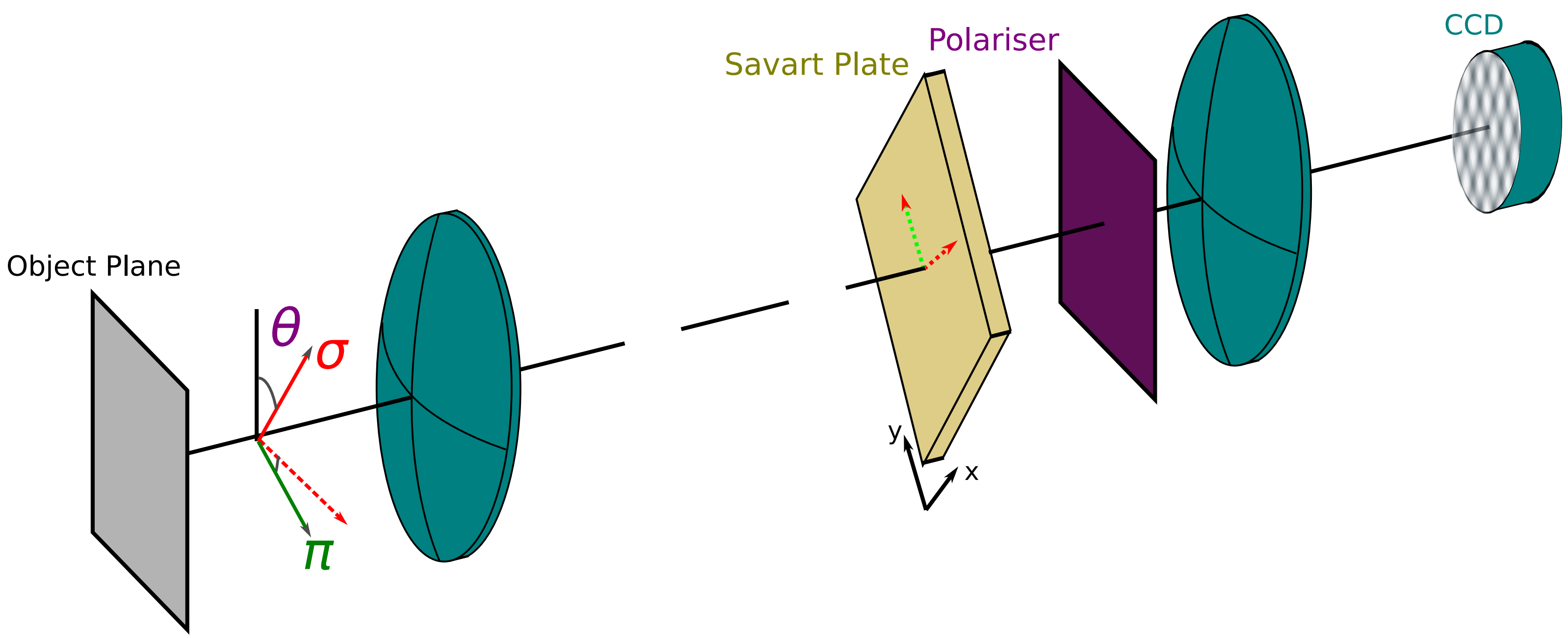
but σ and π are orthogonal.
If they were monochromatic,
they would cancel out...



Spectral Coherence

π and σ orthogonal and always the same intensity, but different spectral profiles.





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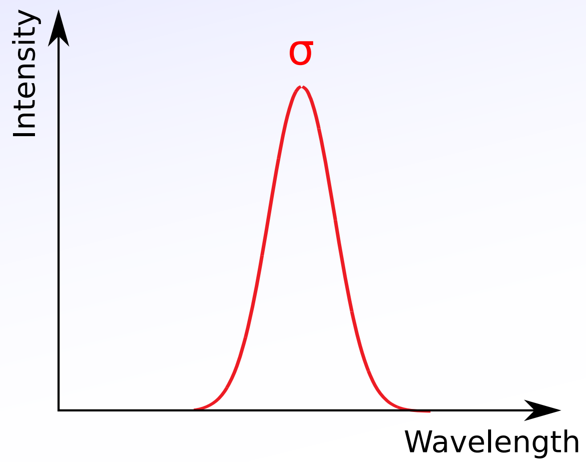
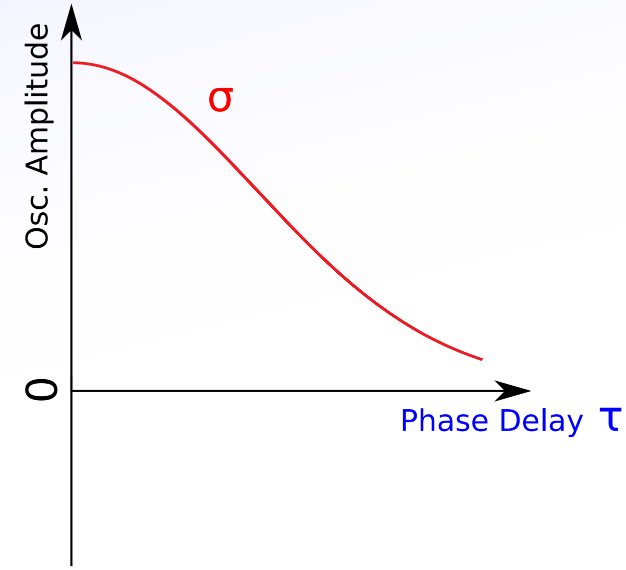
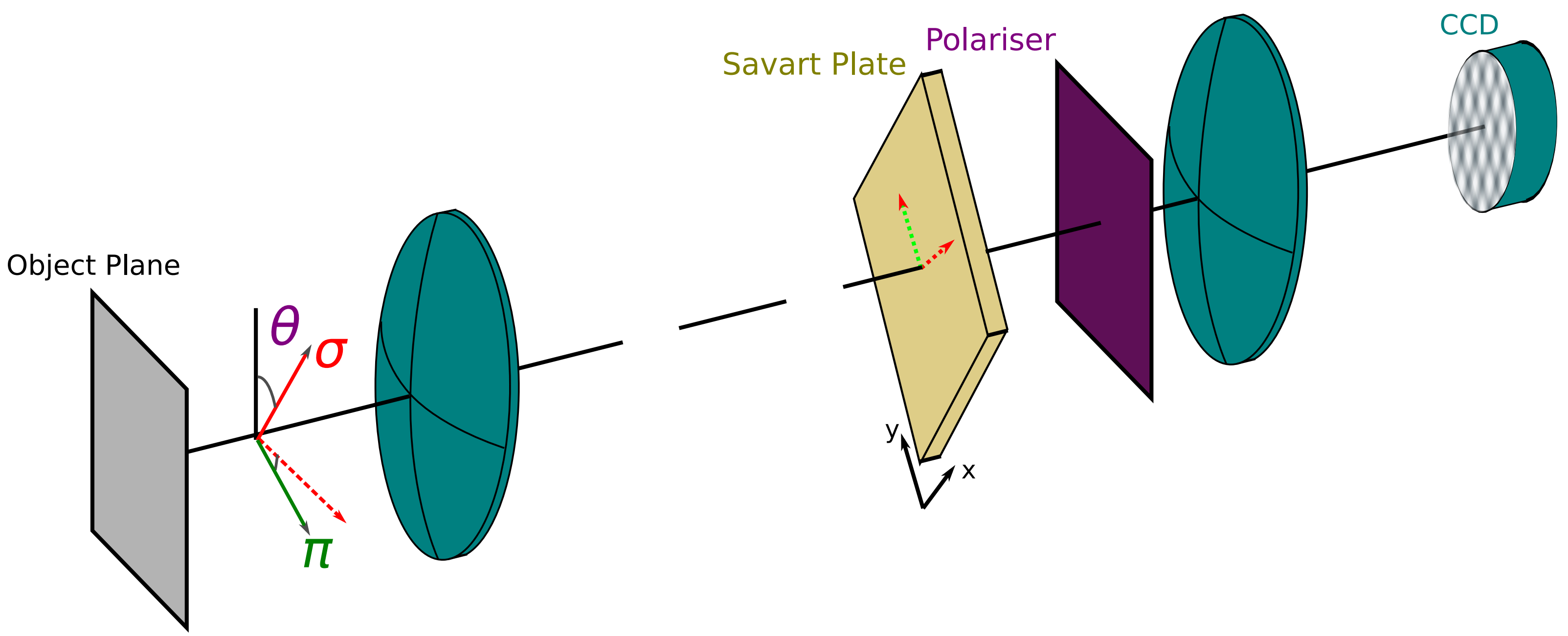
For large τ, different wavelengths have different phases --> decoherence.







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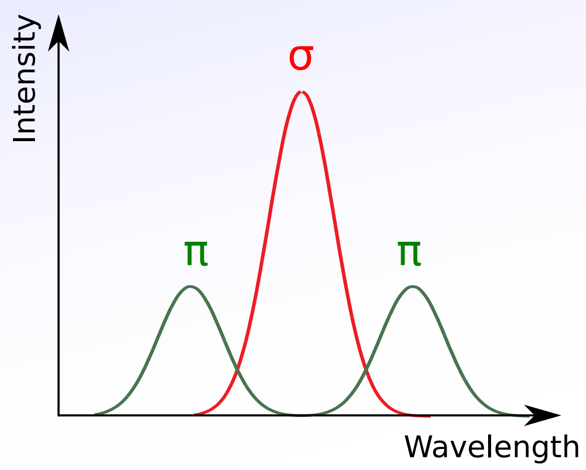

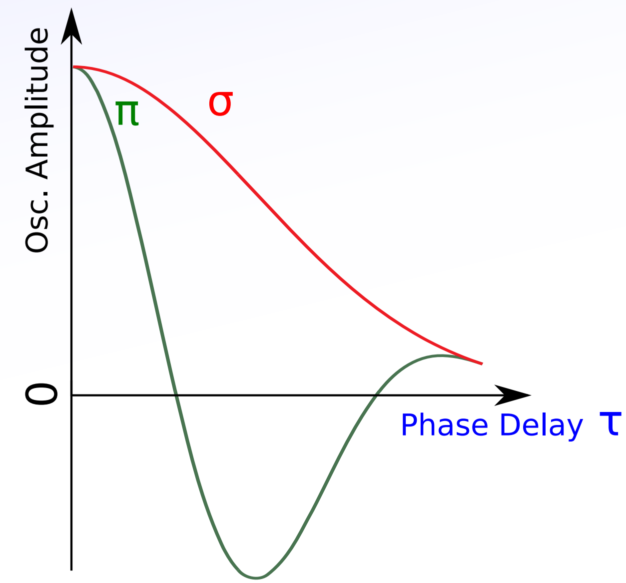

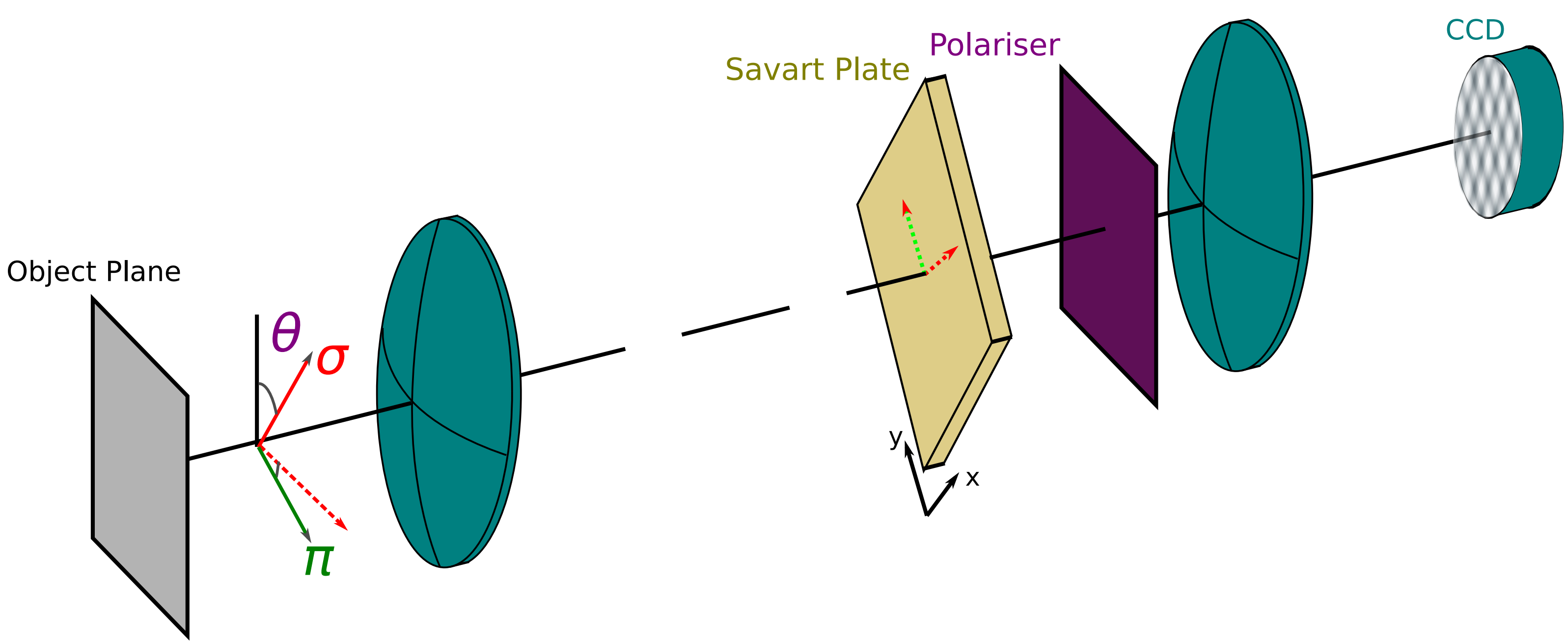

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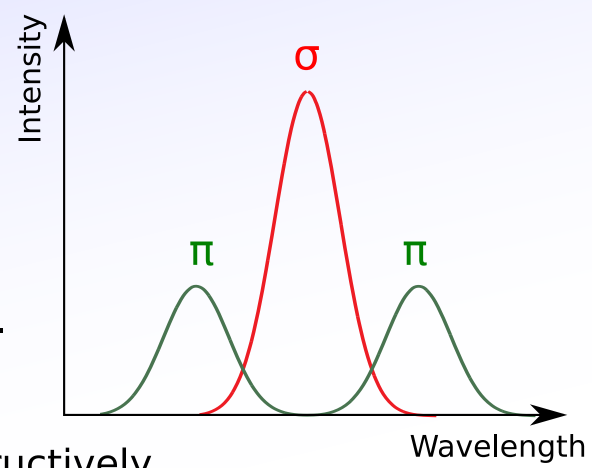
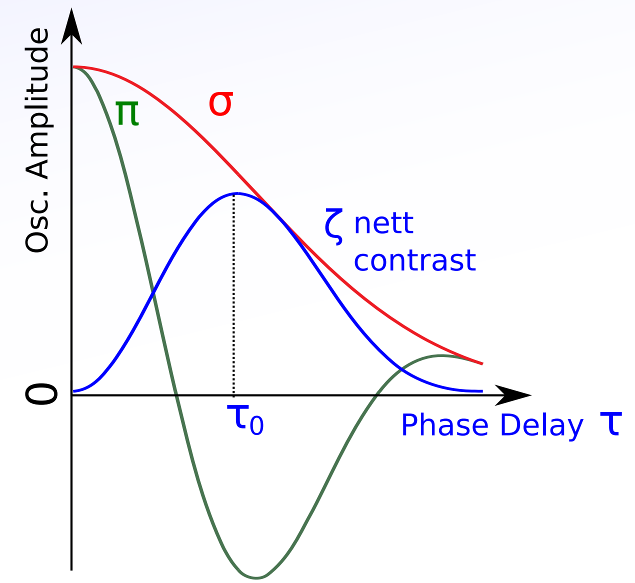
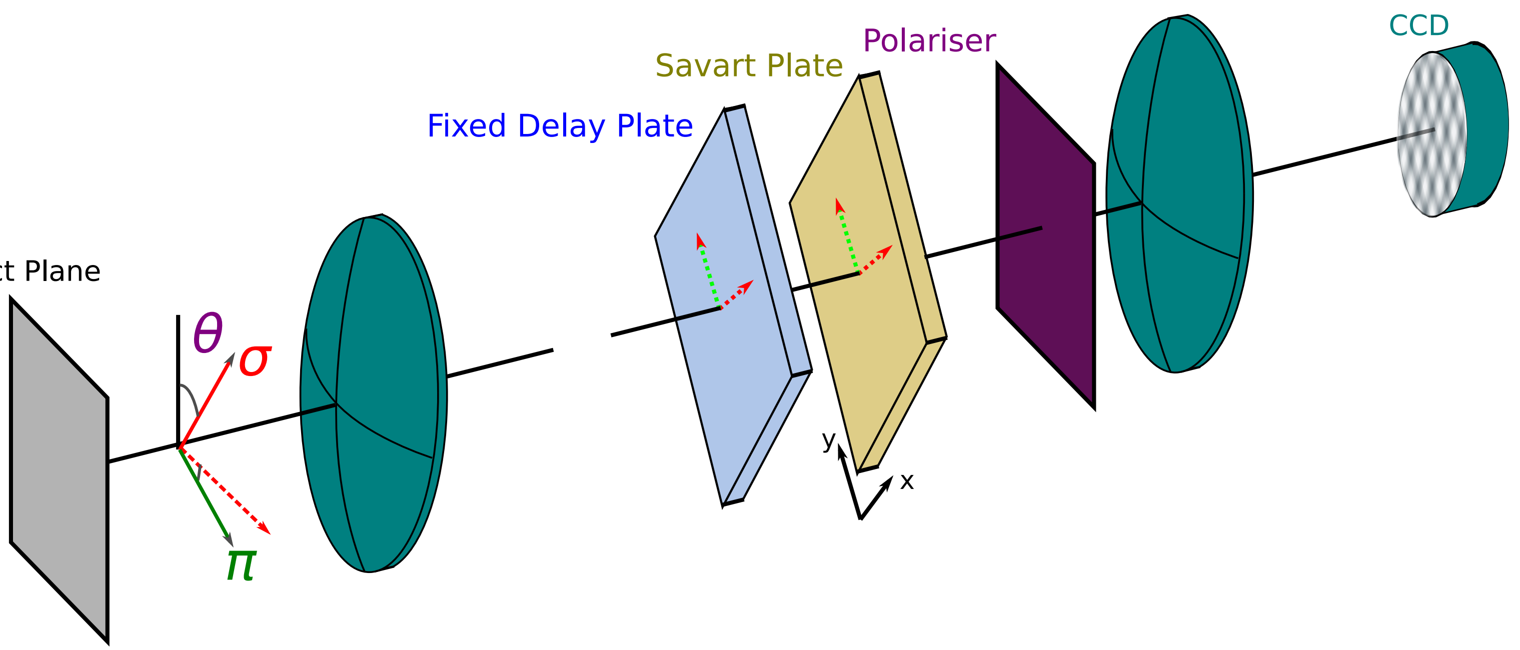




For large τ, different wavelengths have different phases --> decoherence.

Add a delay plate to introduce the best τ_0 - where π and σ combine constructively.



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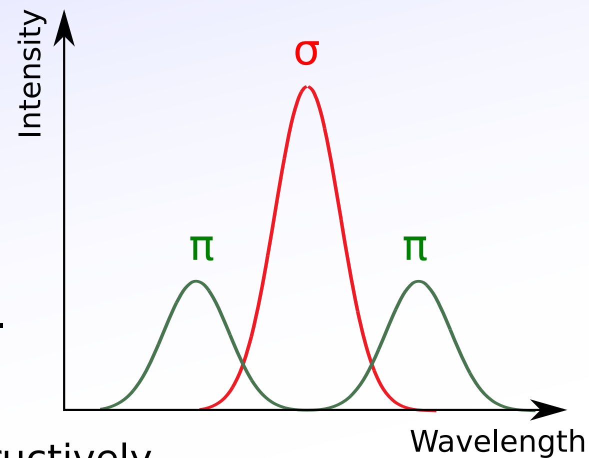

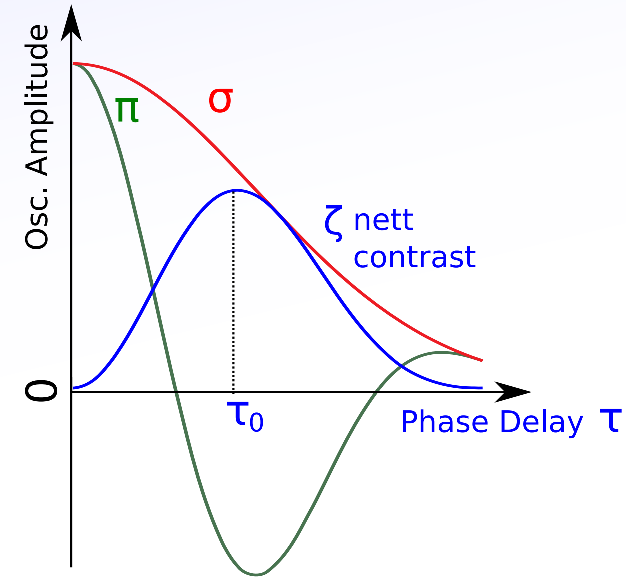

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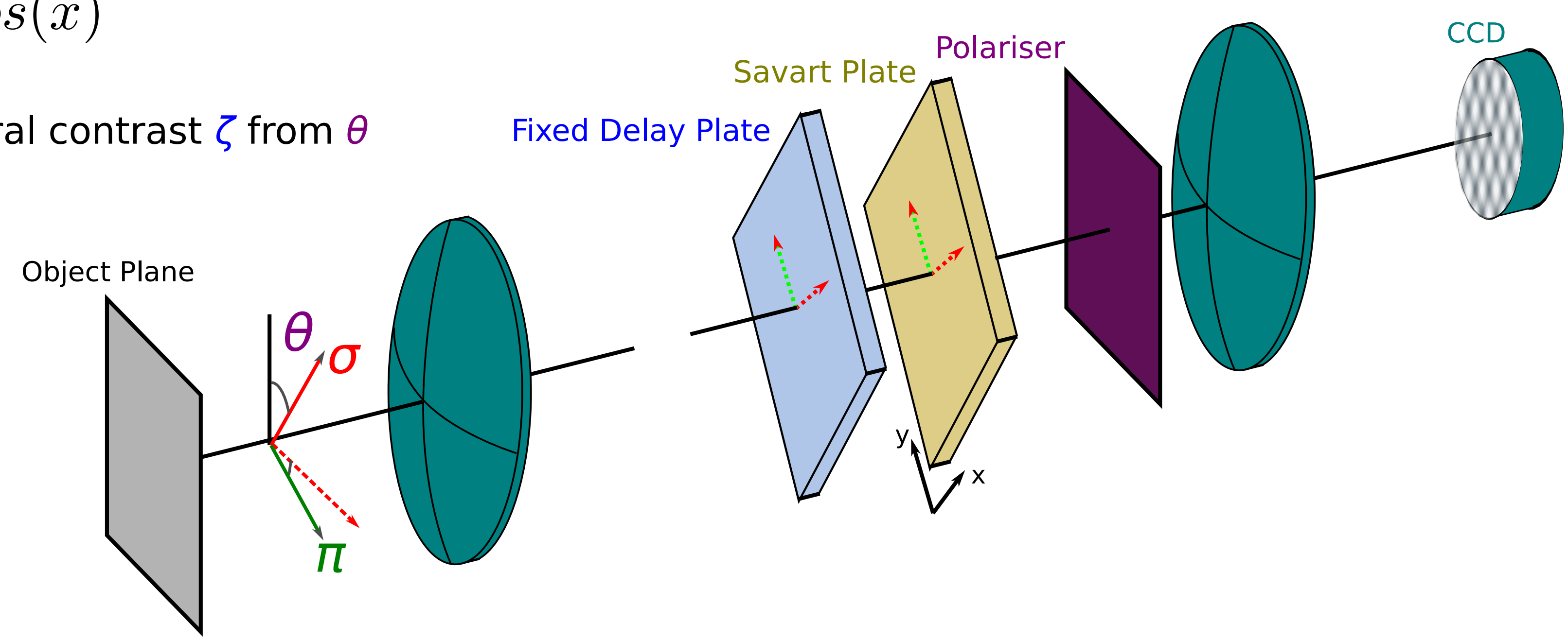
but amplitude now also dependent on contrast:

$$I \propto 1 + \zeta \, cos 2\theta \, cos(x)$$

Need to separate spectral contrast ζ from θ



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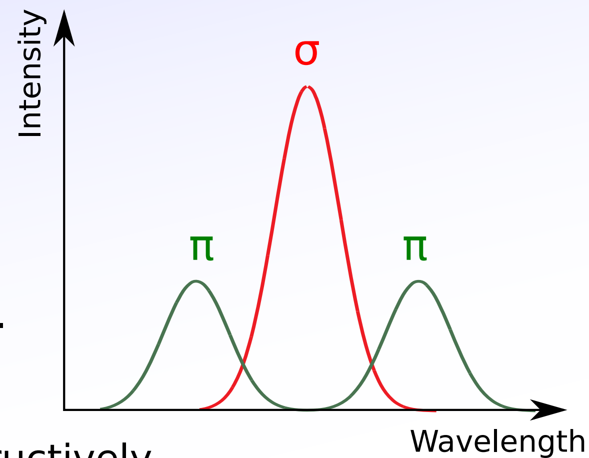

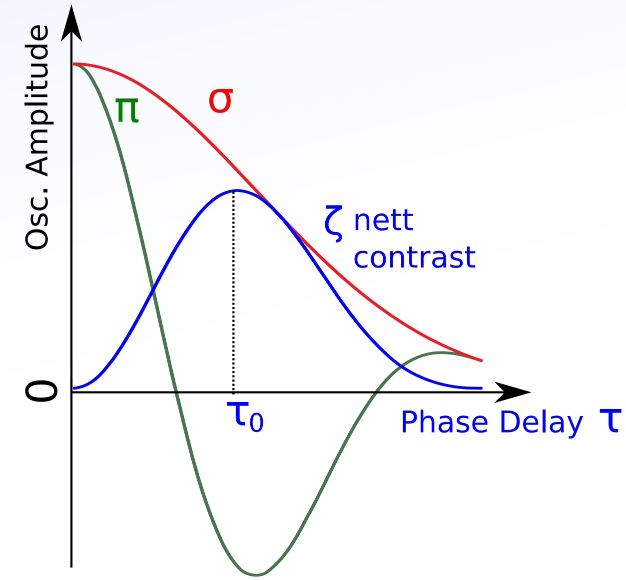

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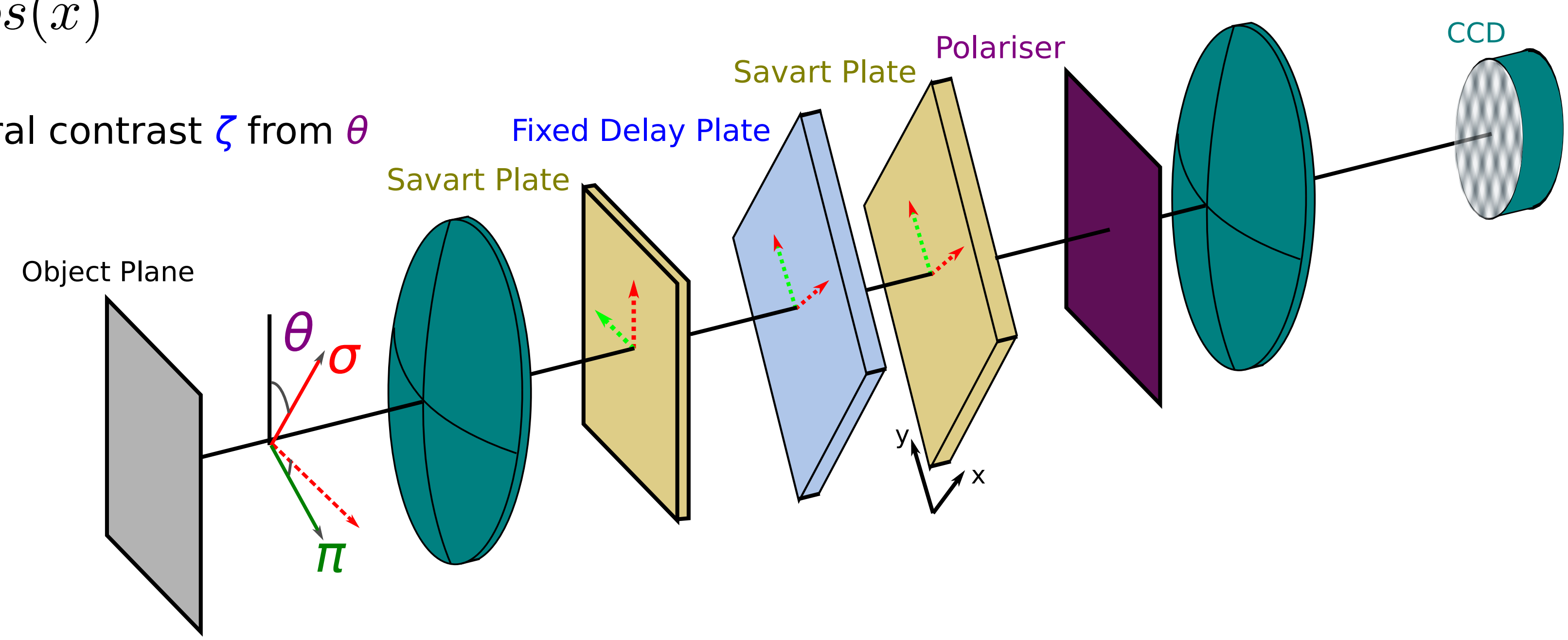
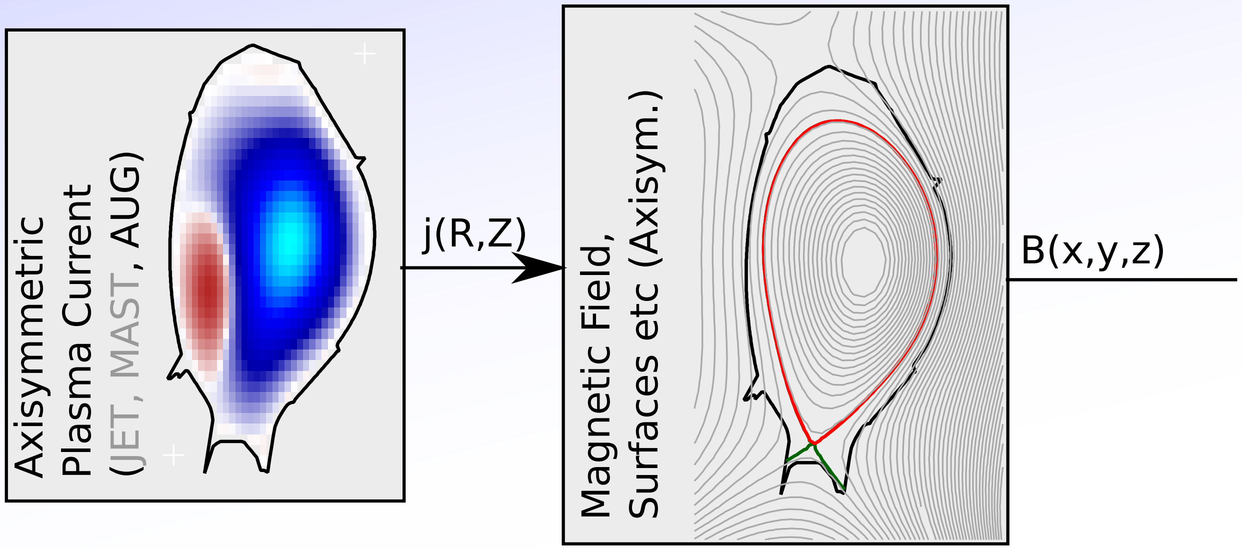
add another Savart plate at 45°.
Combined effect adds 2 extra terms:



$$I \propto 1 + \zeta \; cos2\theta \; cos(x) + \zeta \; sin2\theta \; cos(x-y) - \zeta \; sin2\theta \; cos(x+y)$$

Forward Model

Developed several components for the Bayesian / forward modelling framework (Minerva).



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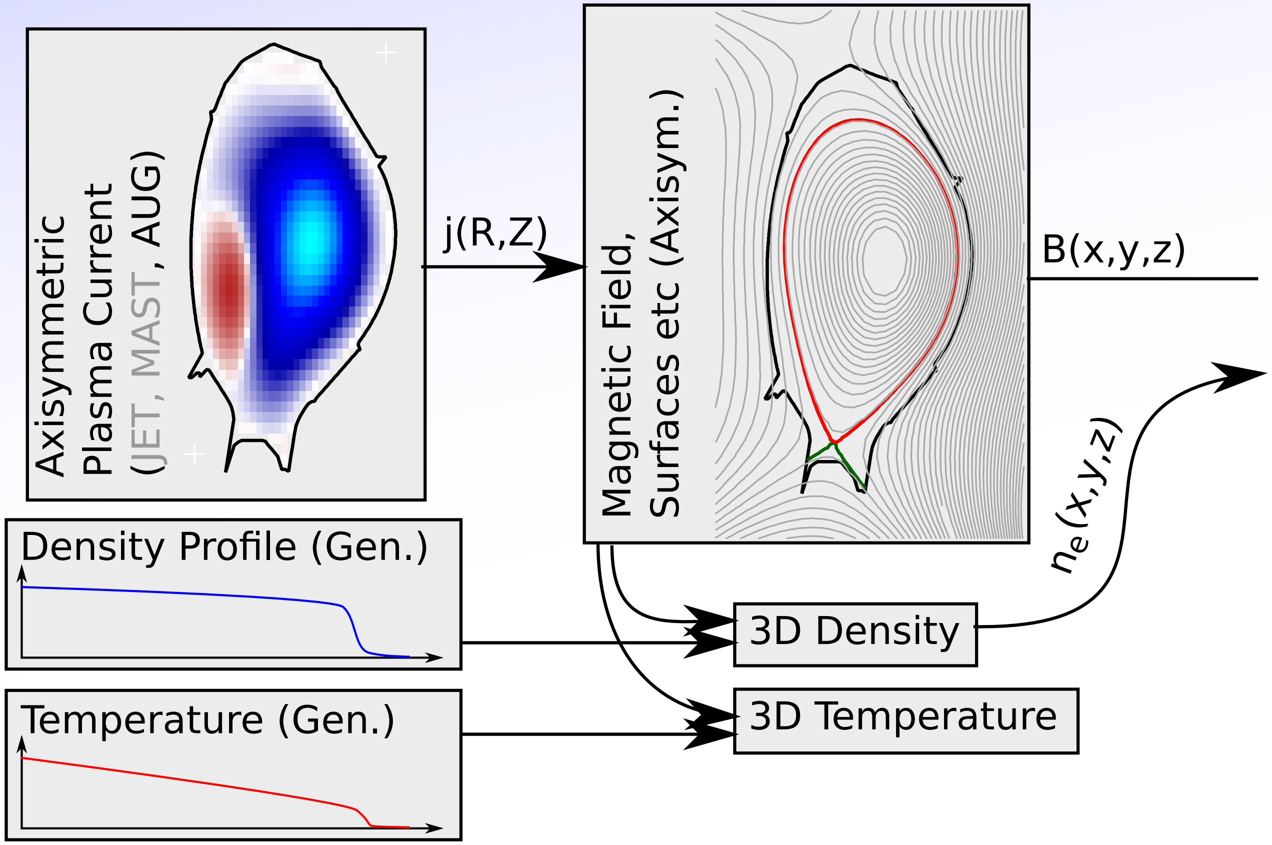

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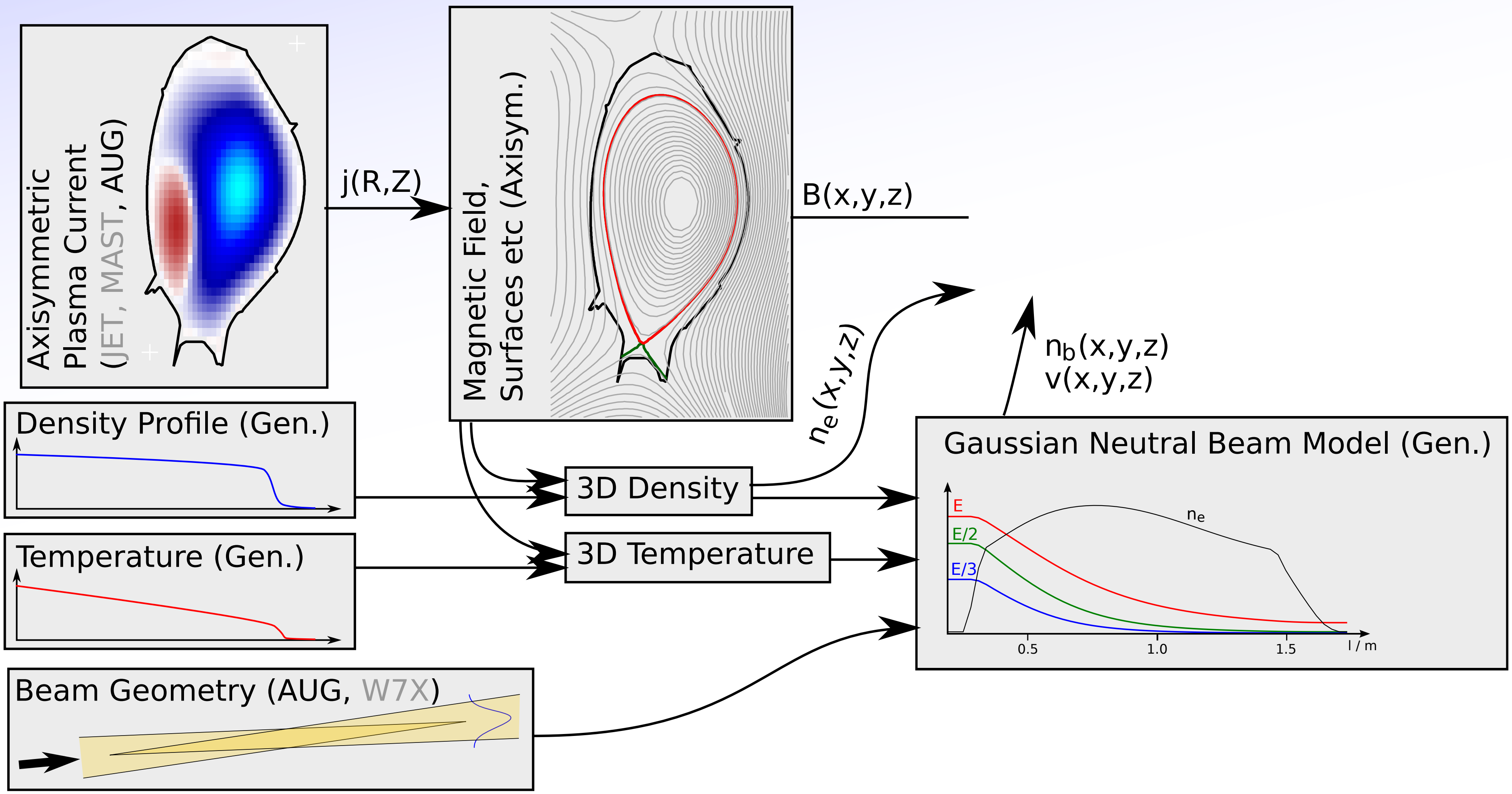

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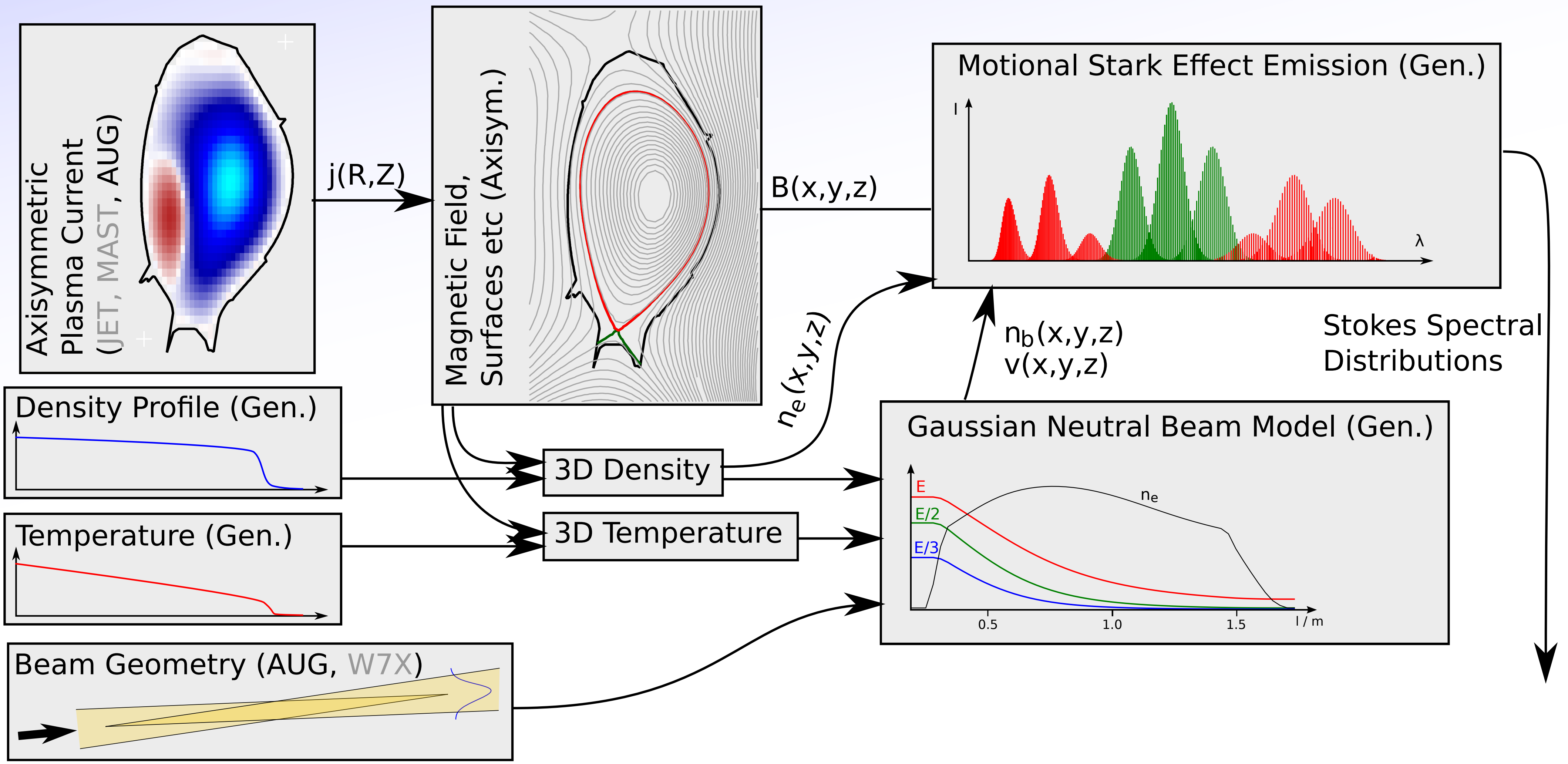

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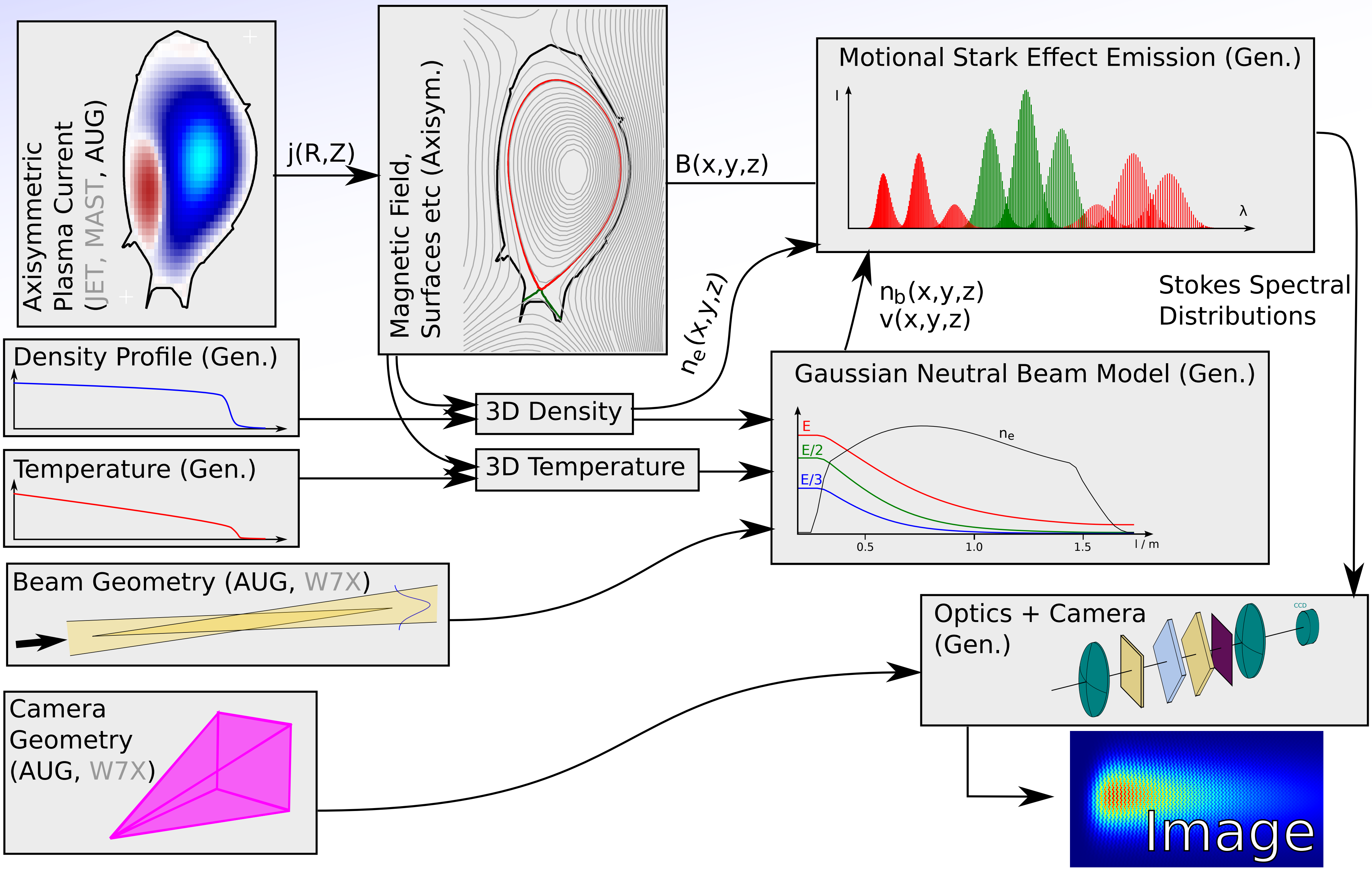
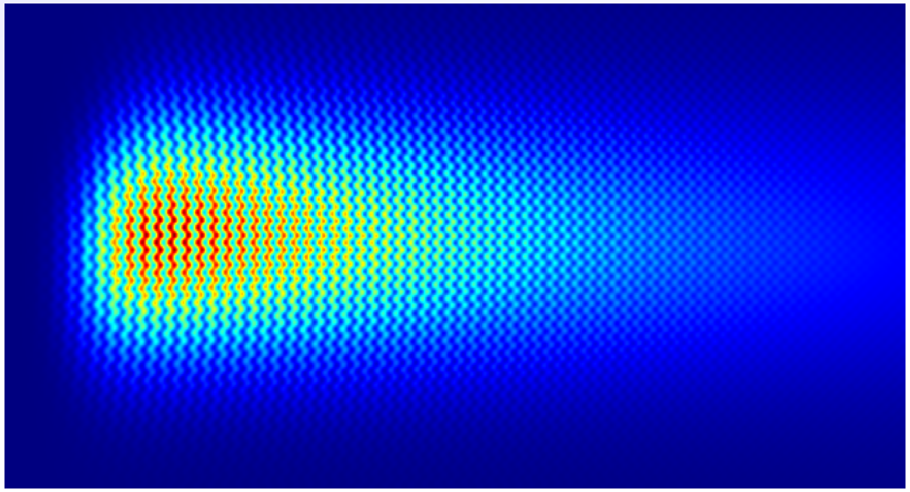


Savart Plates

(For the record: This is the '*Ampltiude Modulated Double Spatial Hetrodyne*' system).



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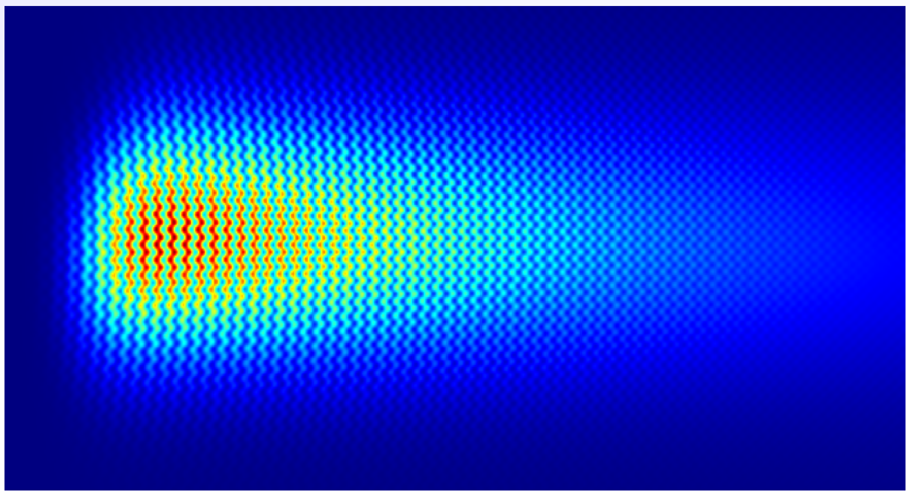
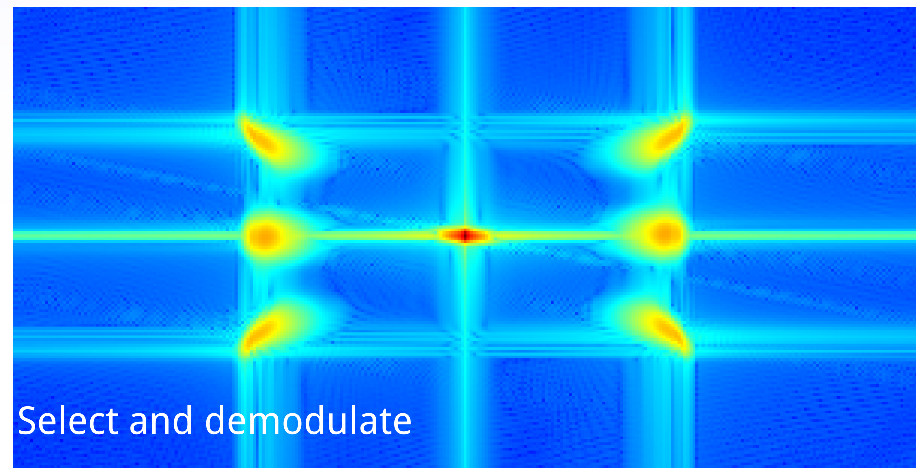
$$\frac{I}{I_0} = 1 + \zeta\, cos2\theta\, cos(x) + \zeta\, sin2\theta\, cos(x-y) - \zeta\, sin2\theta\, cos(x+y)$$

Contrast

Polarisation Angle



FT →



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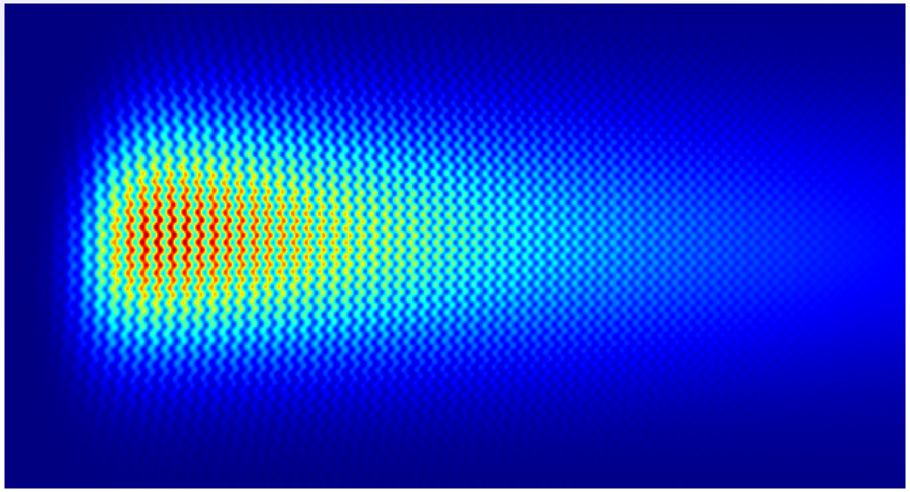

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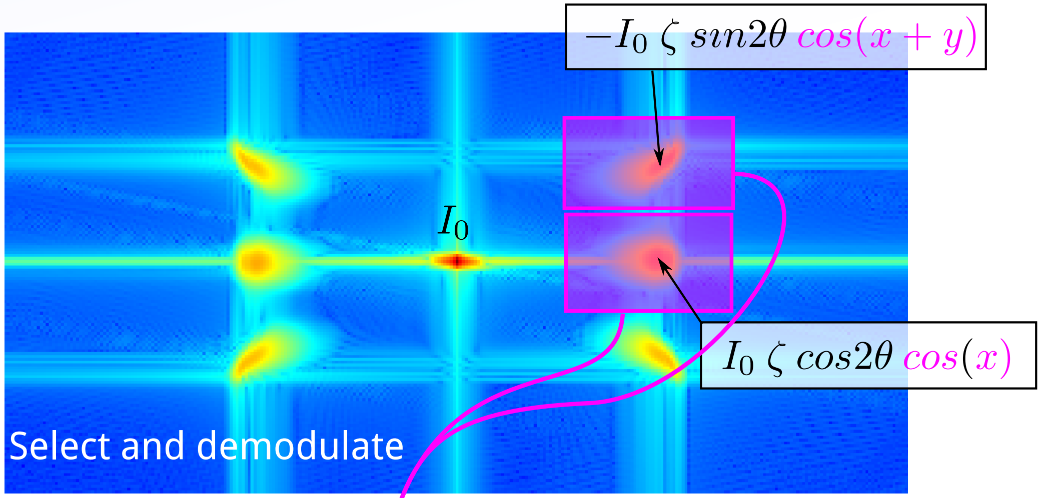

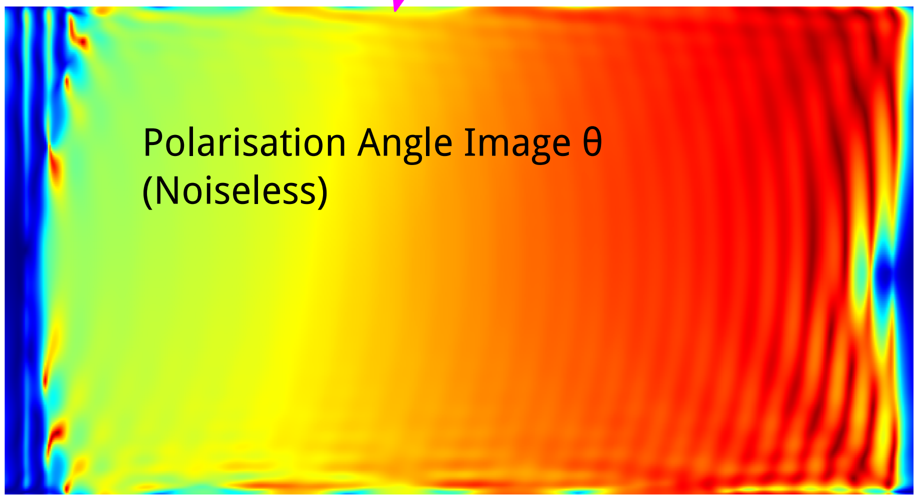

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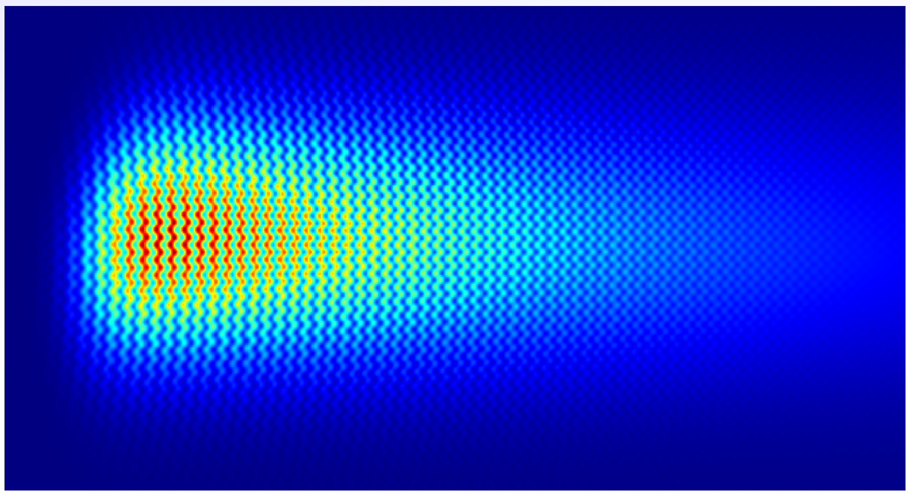
Contrast   ϕ?   Polarisation Angle   ϕ?   ϕ?



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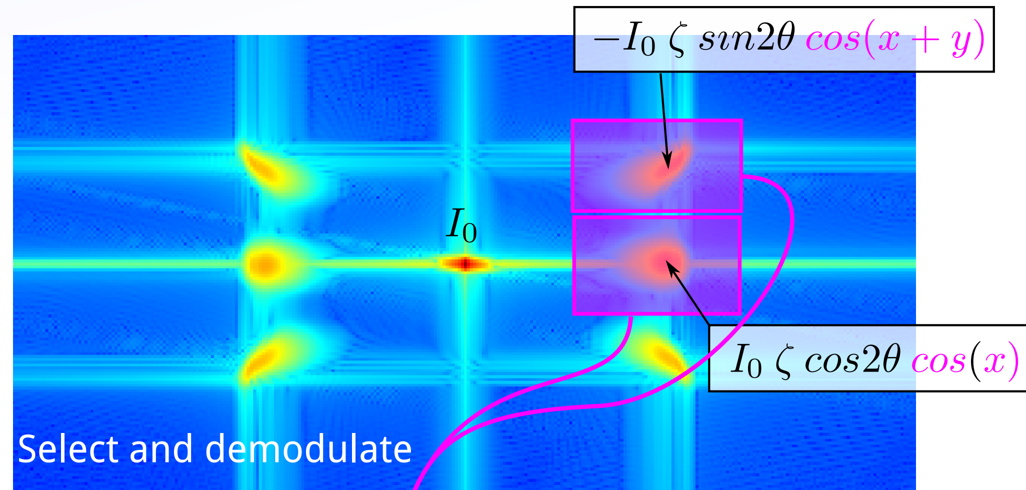

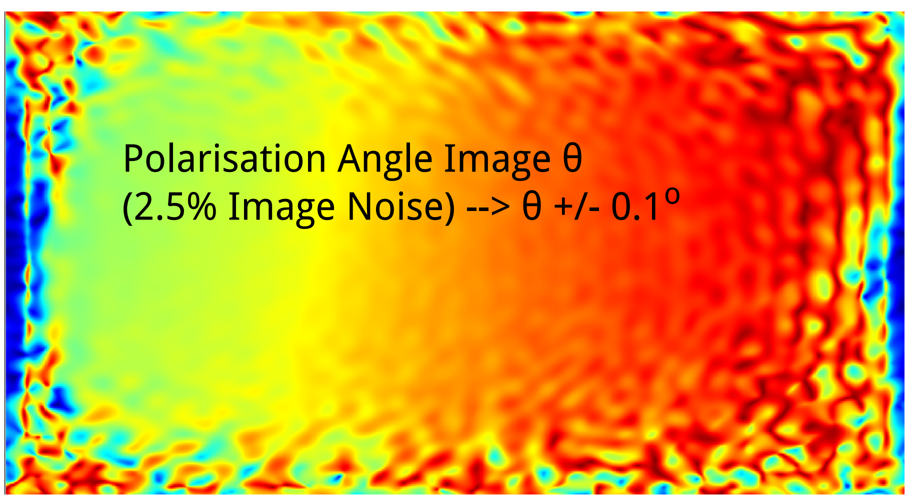

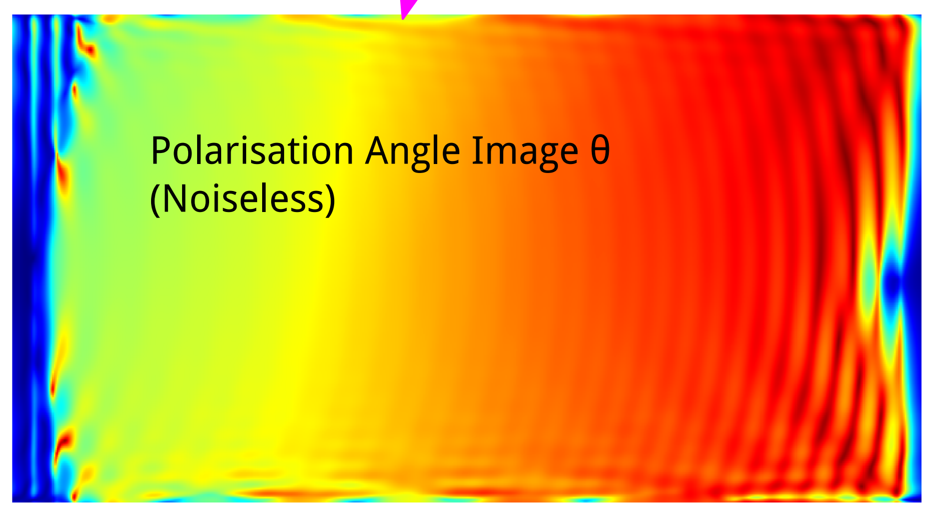

Demodulated θ matches LOS average (with some extra phases).

Recovery of plasma current - Axisymmetric

Final objective is plasma current - *j(R, Z)*.
Normally θ used as a constraint for equilibrium.
With 2D measurements, can we calculate *j* without equilibrium?

Assuming toroidal symmetry:

$$-\mu_0 j_\phi = \frac{\partial B_z}{\partial R} + \frac{1}{R}\frac{\partial^2}{\partial Z^2}\int_0^R R' B_z(R', Z)\, dR'$$

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This we have
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Function of Z that we still cannot know.

The new term gives localisation of current in Z (~via curvature of field).

~10% of j_ϕ and measurement too noisy.

No: It still cannot be directly calculated.

However, we gain dBz/dR at different Zs.
Complex tomography problem, but we may not need equilibrium...

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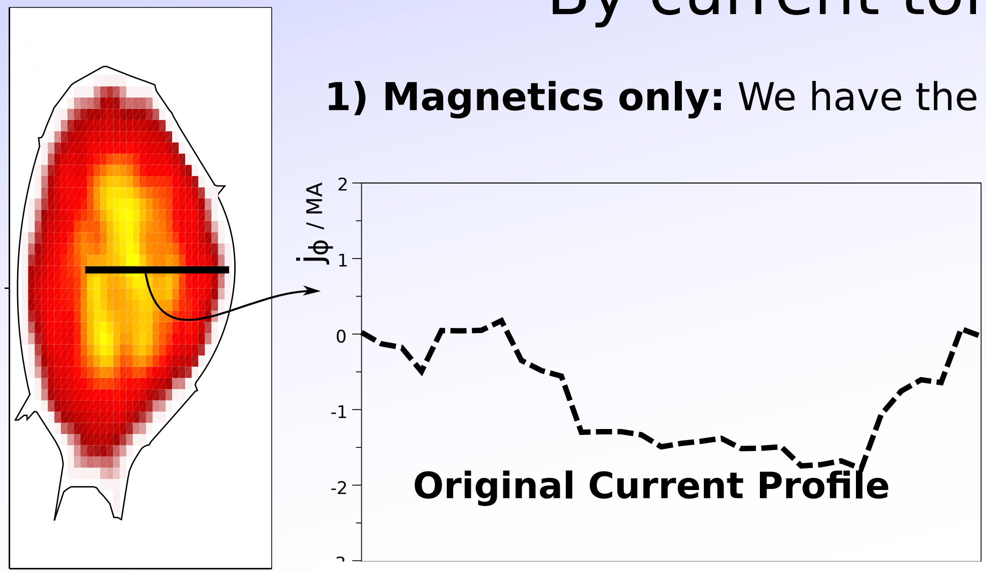

Each case has 900 measurements at *σ = 10mT.* So difference is only in the ***type*** of information.

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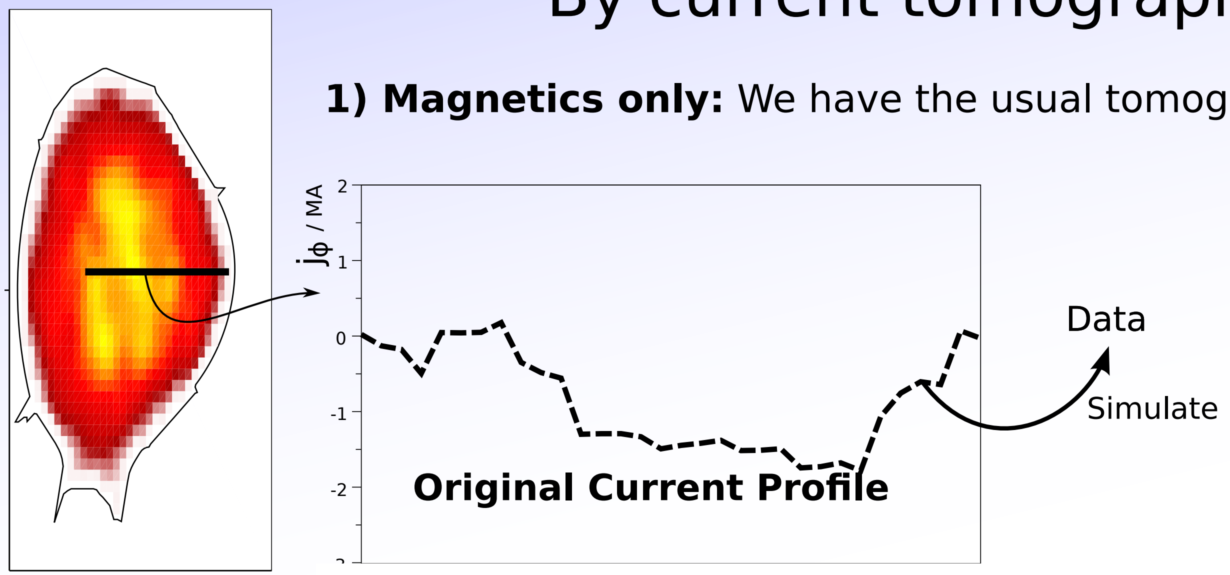

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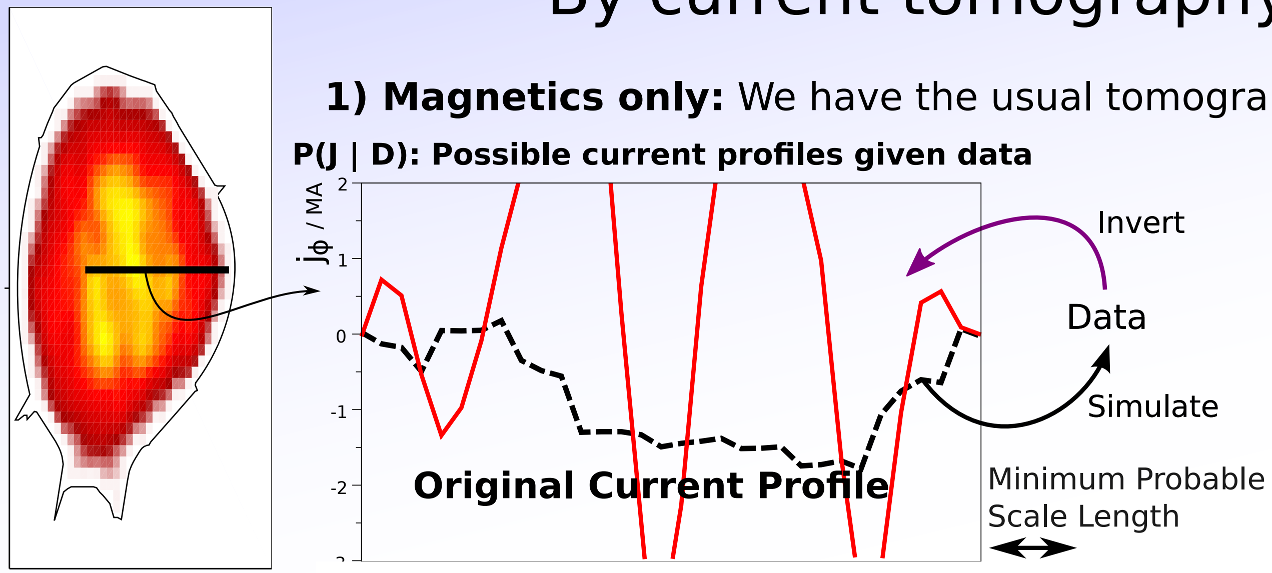

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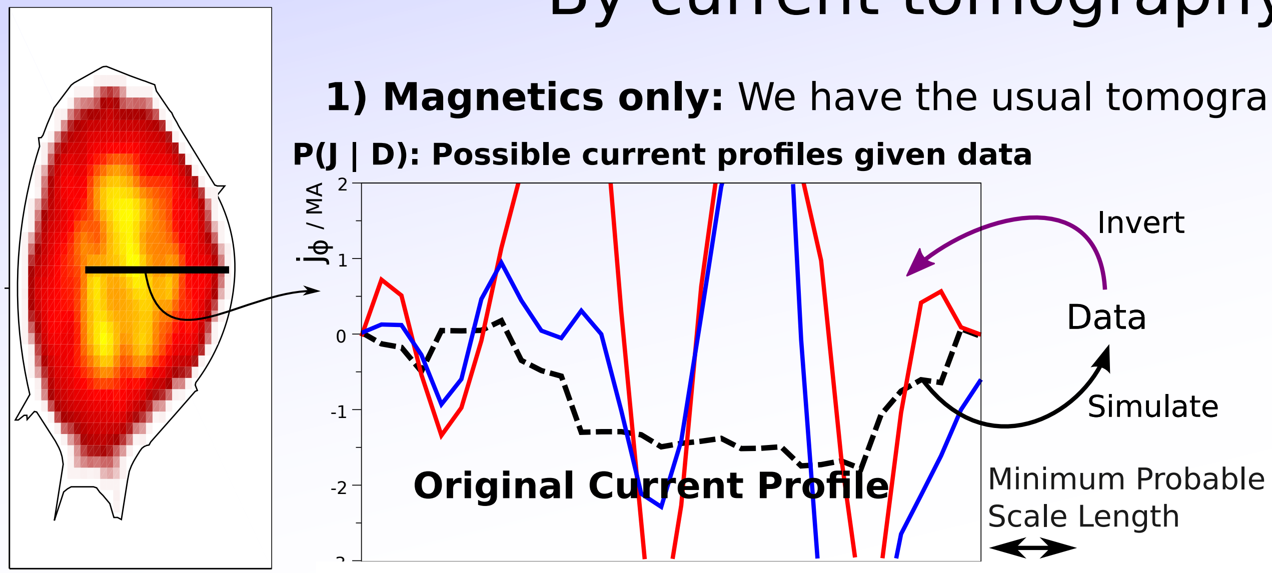

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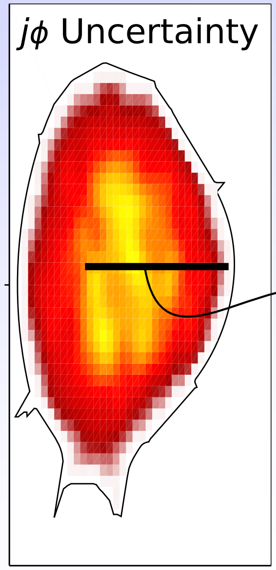
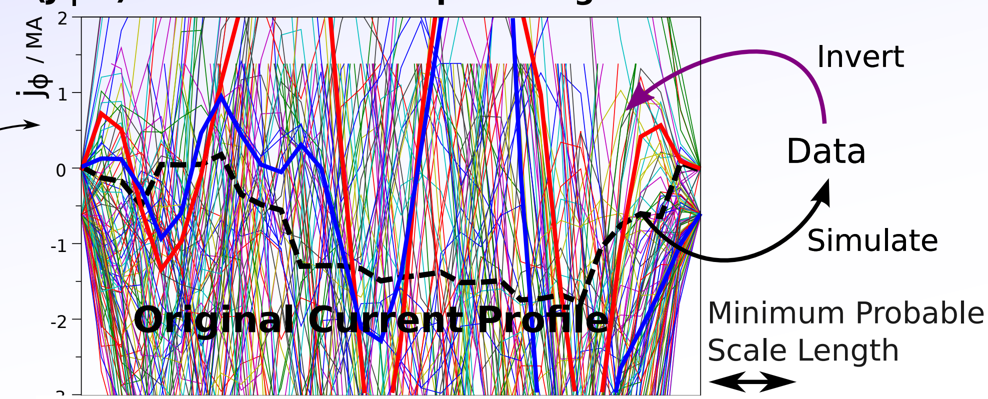


P(J | D): Possible current profiles given data



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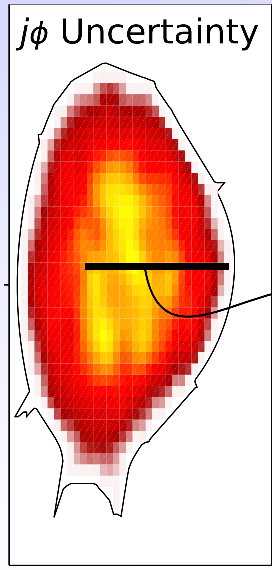

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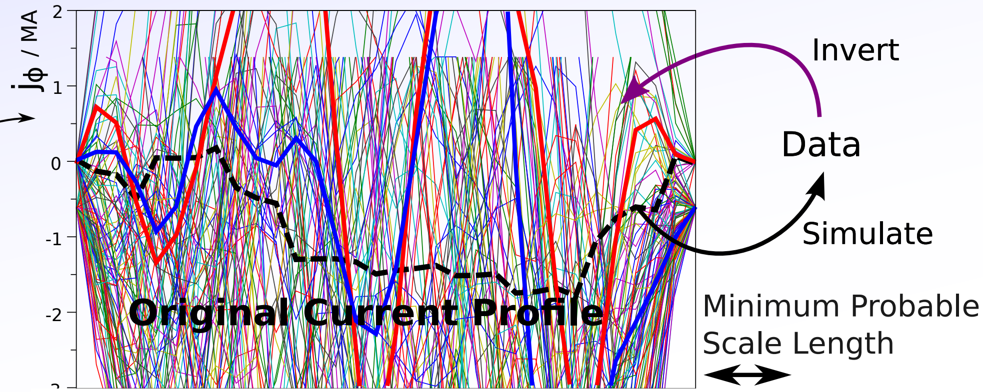

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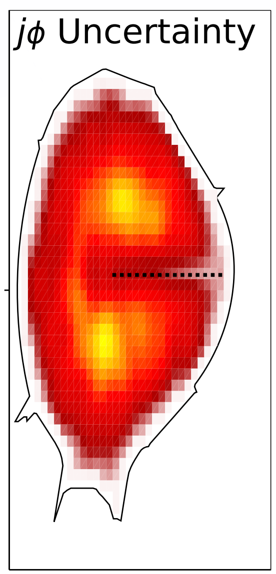
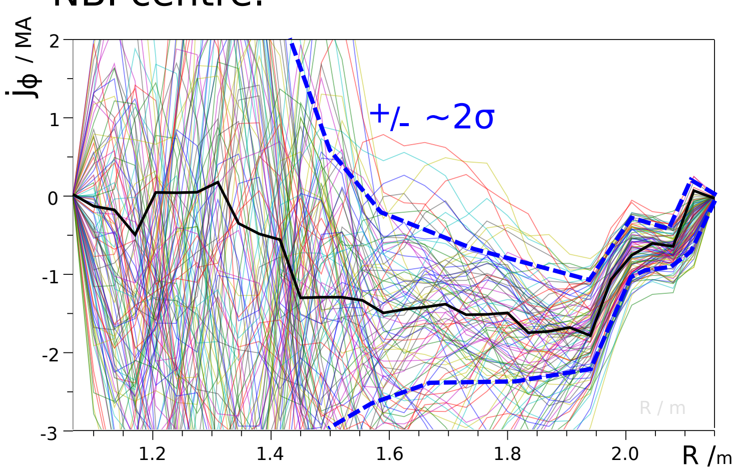


2) +Normal MSE system:
30 x *Bz* at 30 positions along NBI centre.



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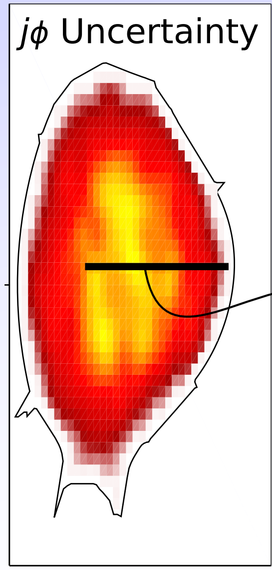

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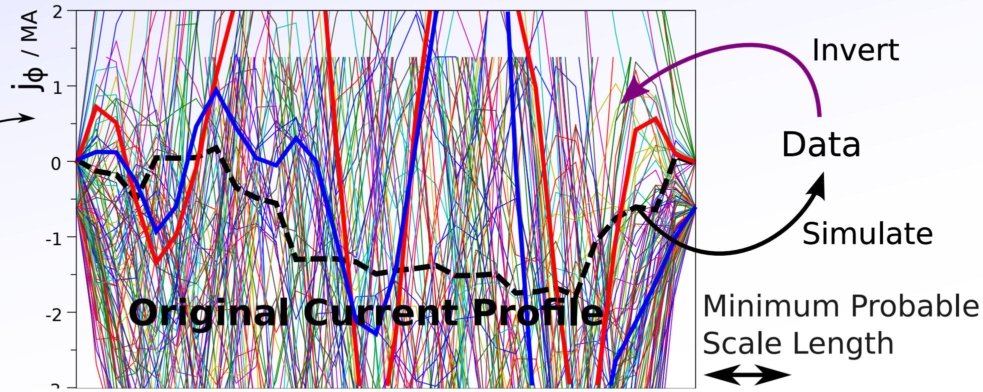

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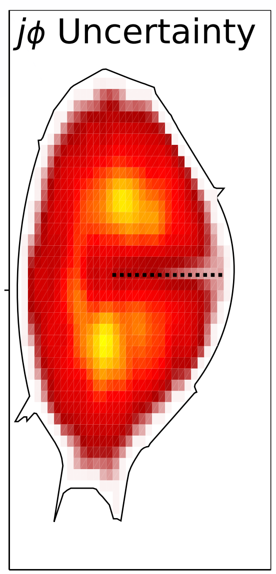

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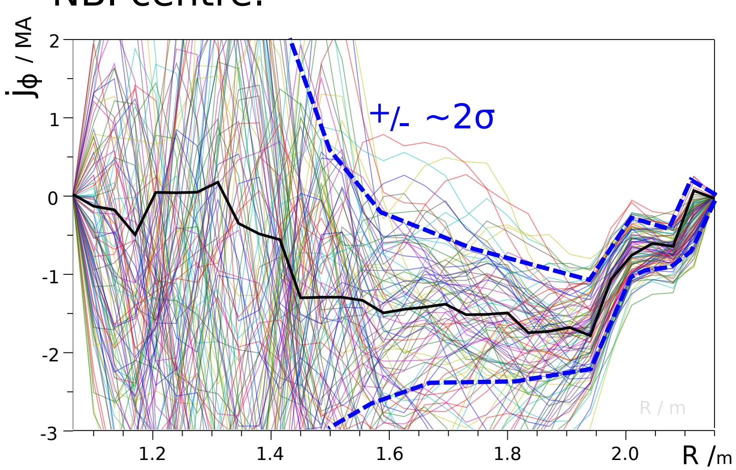

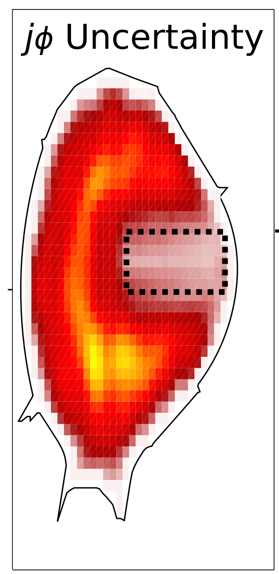

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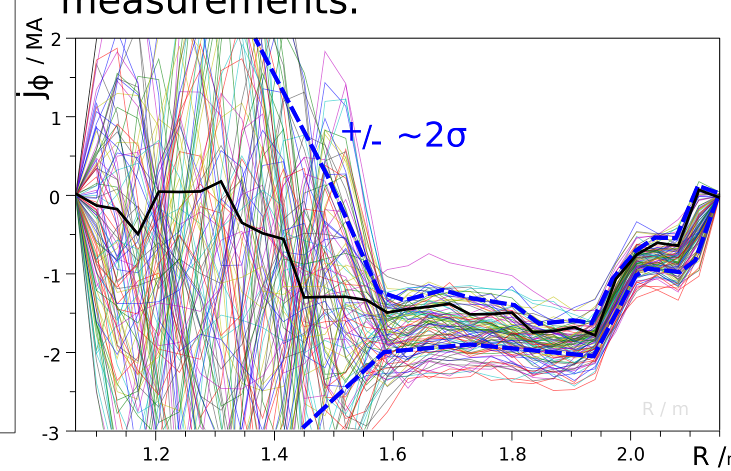

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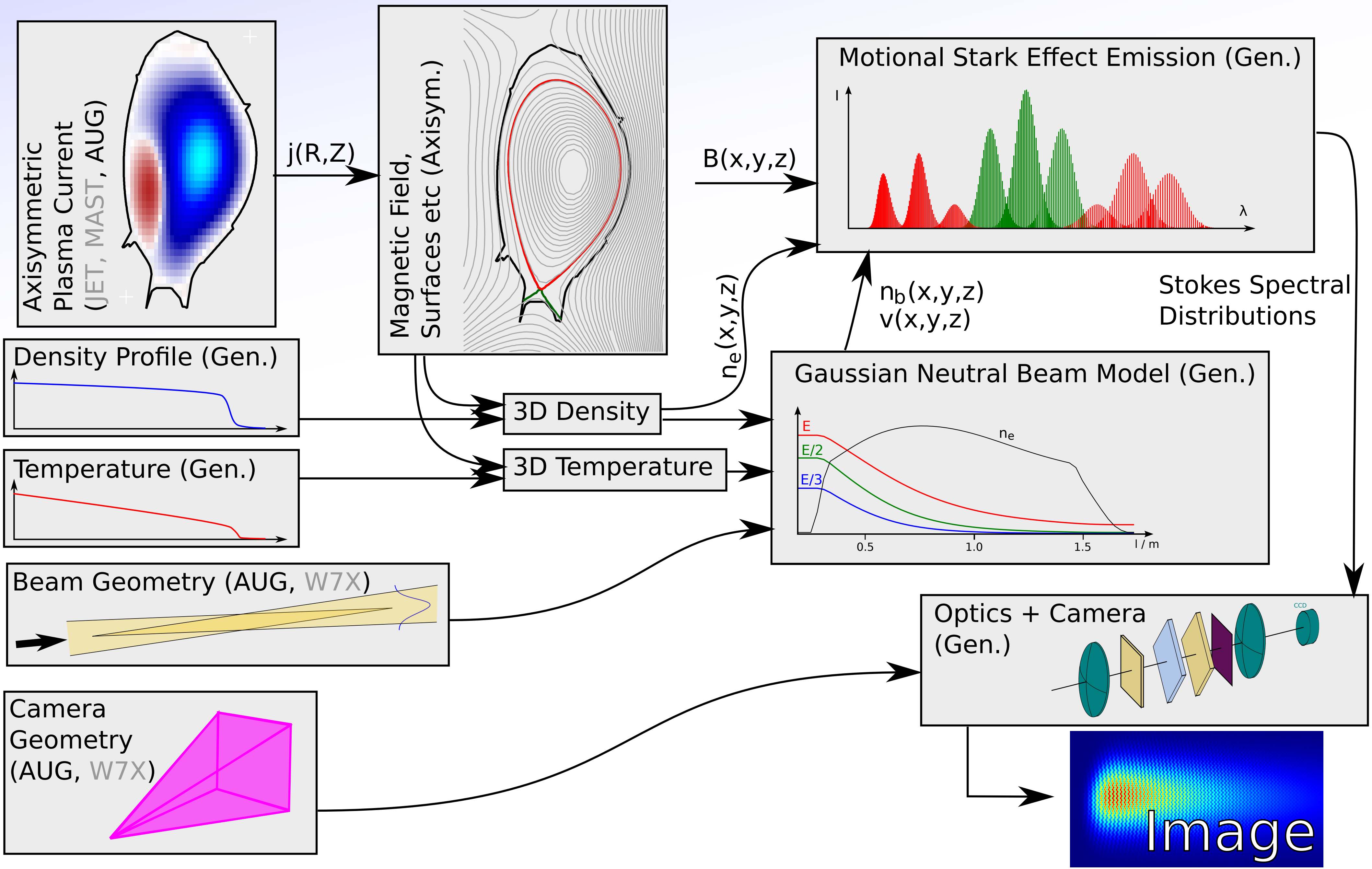
Conclusion: 2D information greatly improves current inference ability, even *excluding* increase in data quantity.

Forward Model (W7X)



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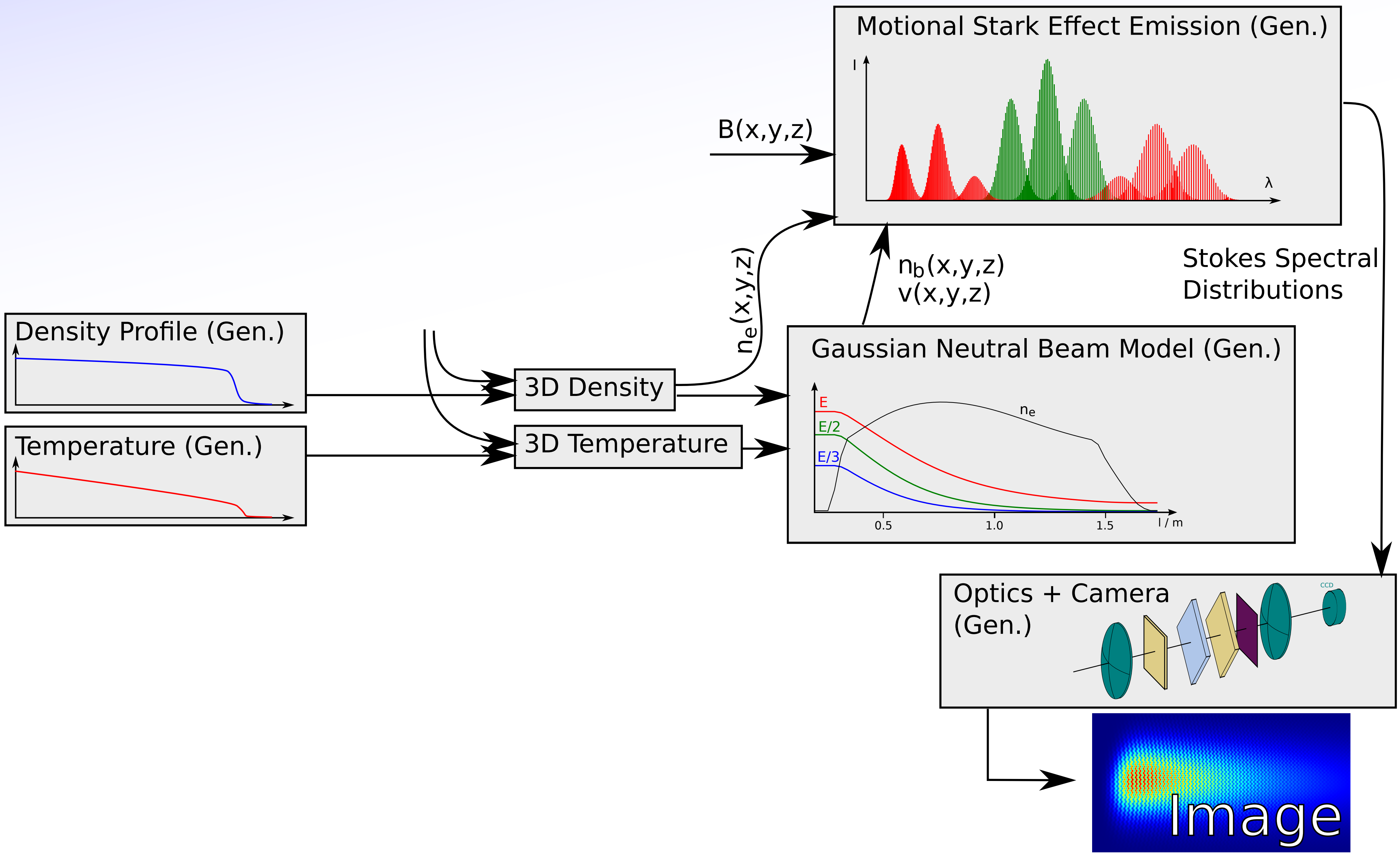

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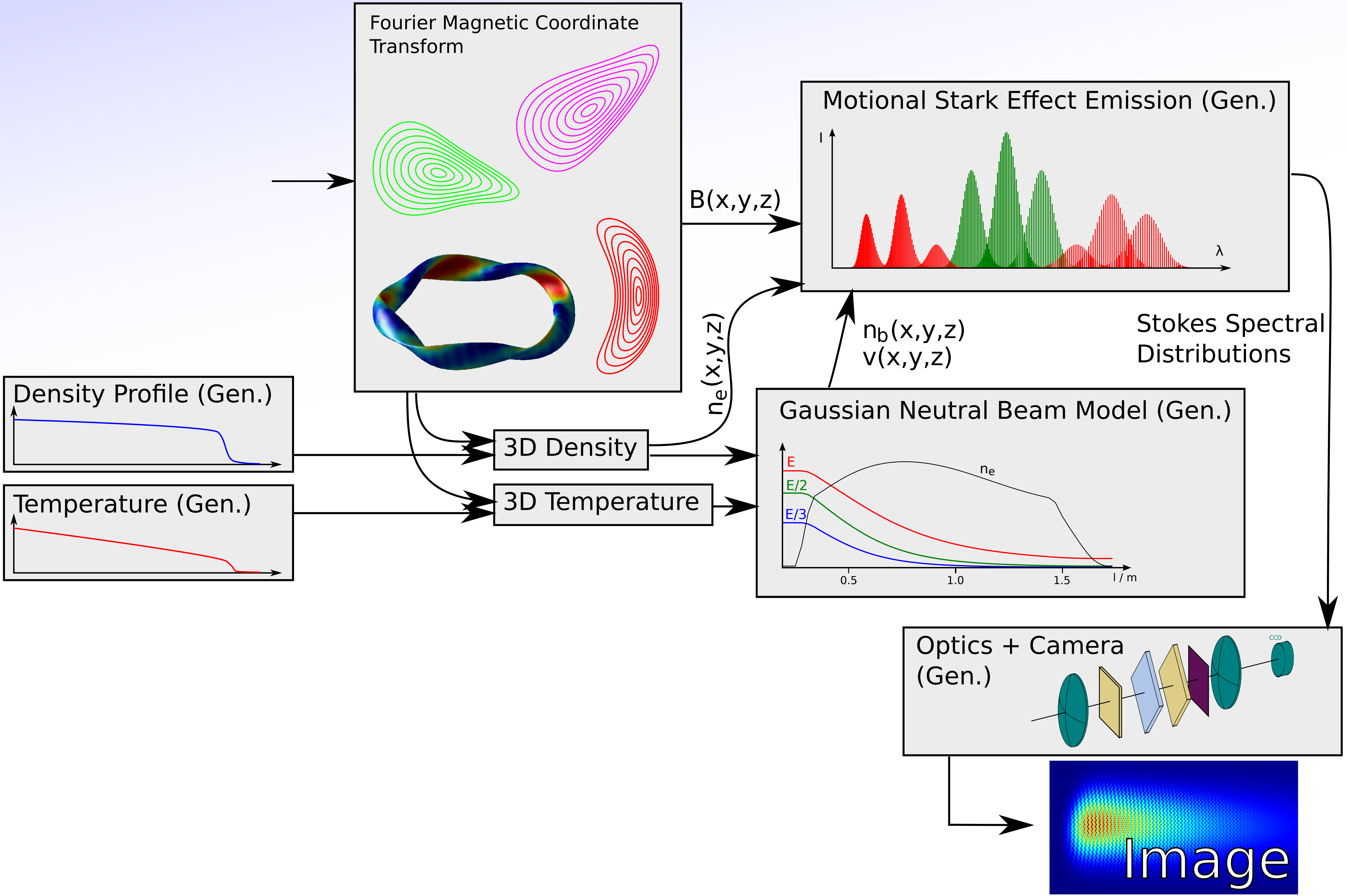

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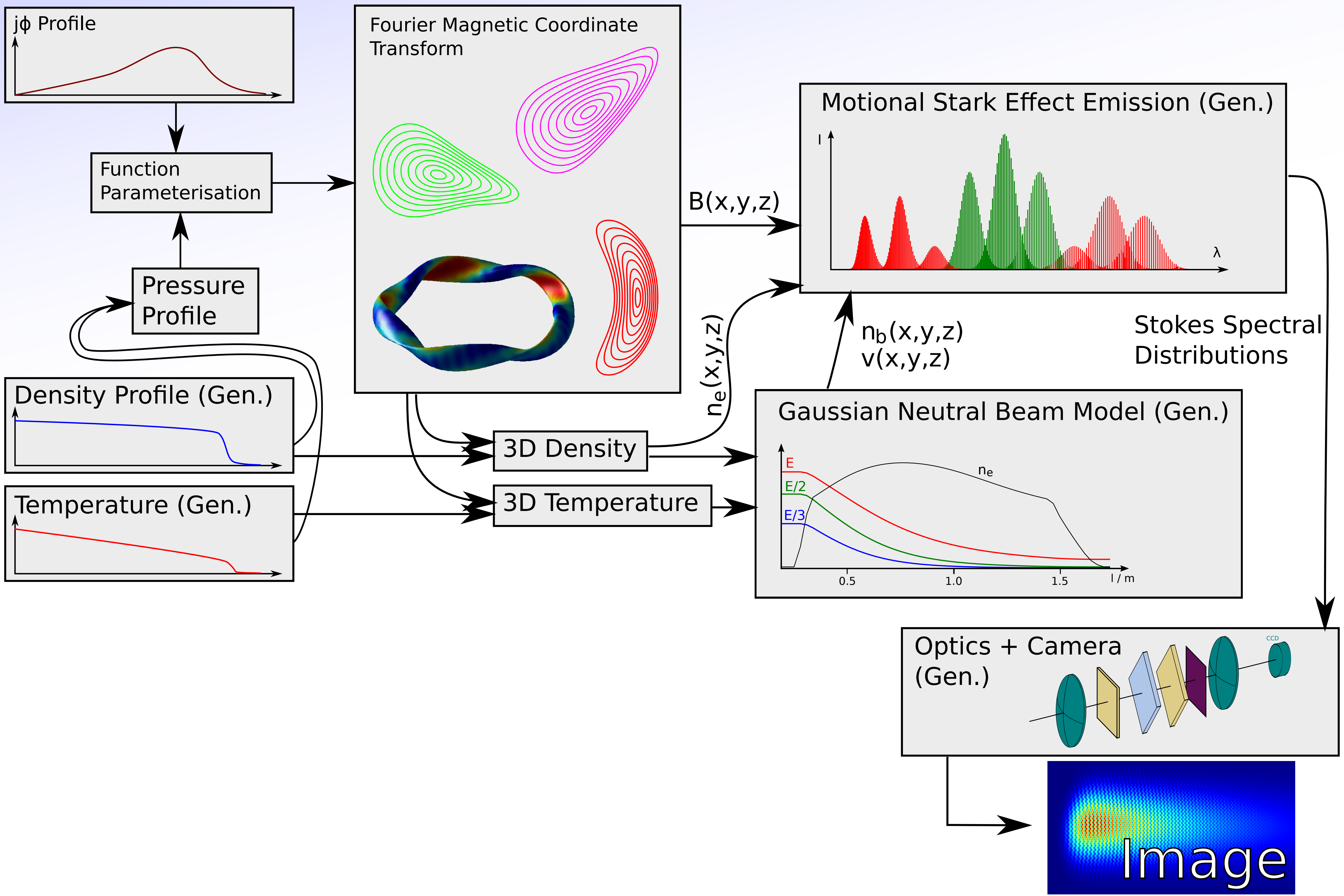

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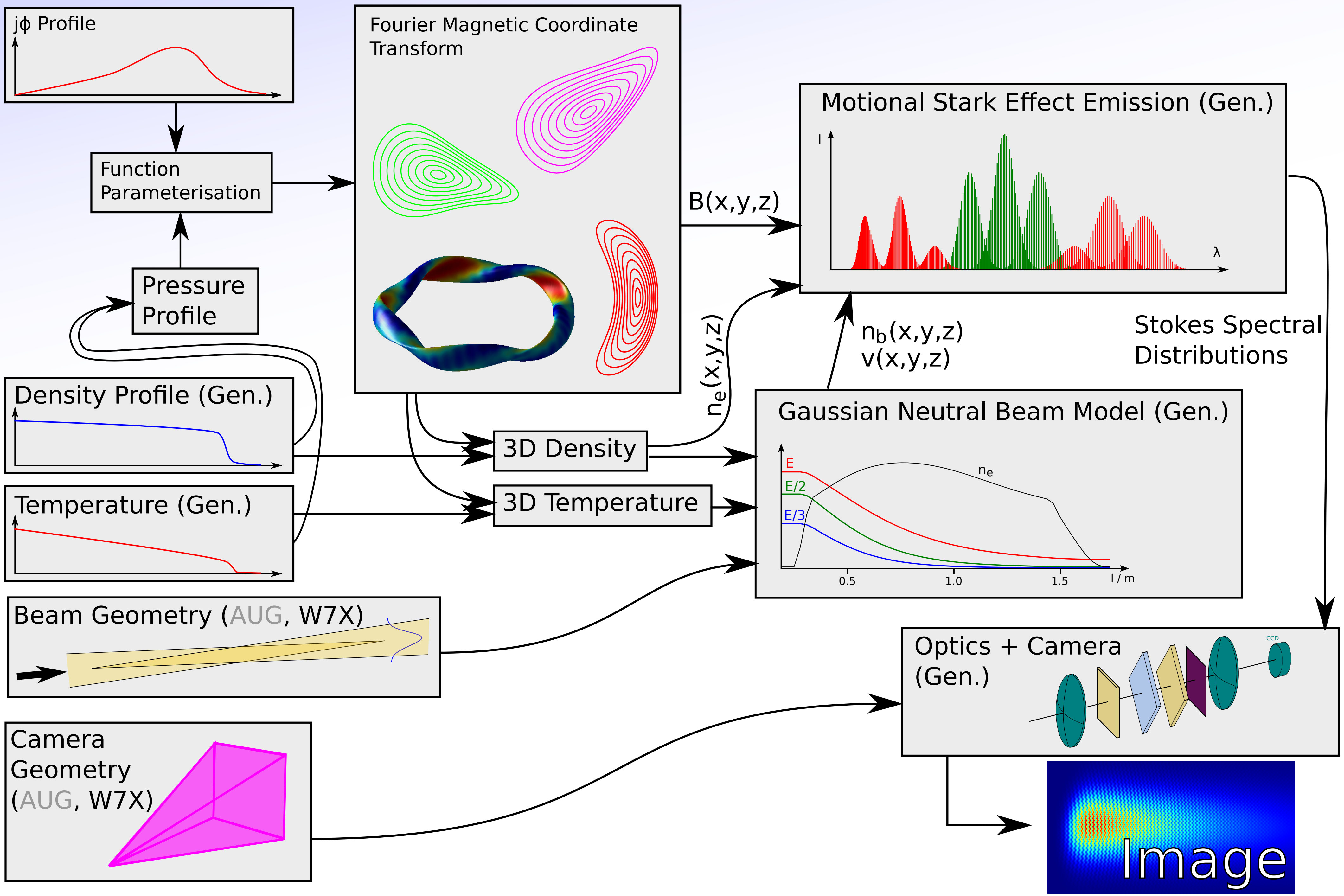

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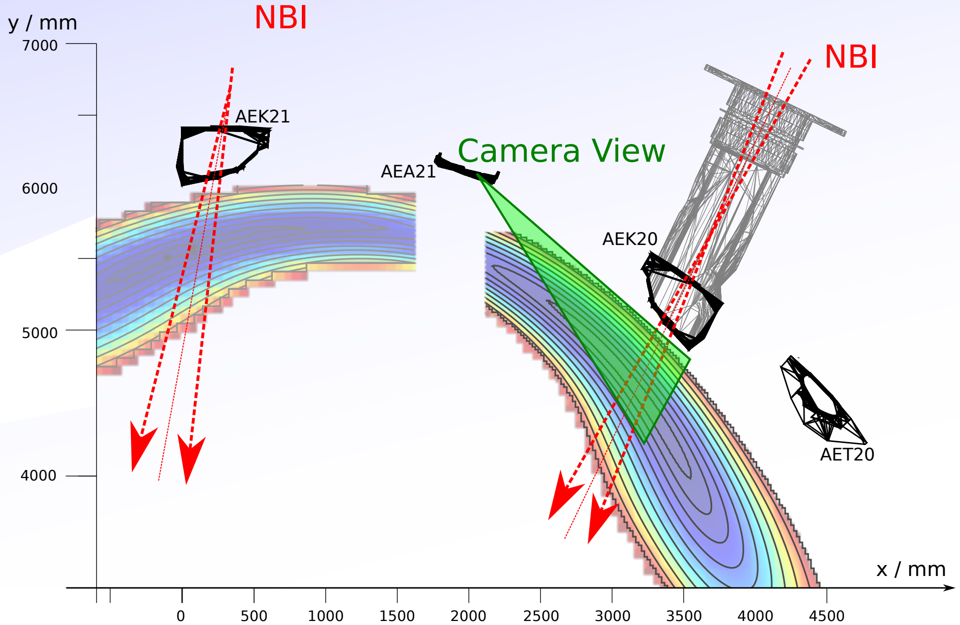
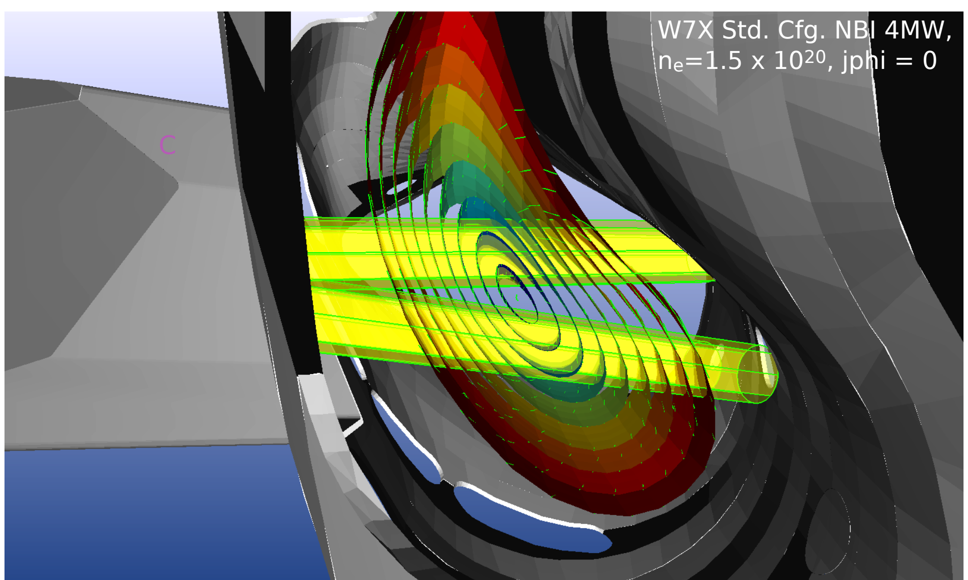
Best view of NBI to reduce cross-surface integration.
More tangential lower beam gives best Doppler shift --> Better image fringe contrast.





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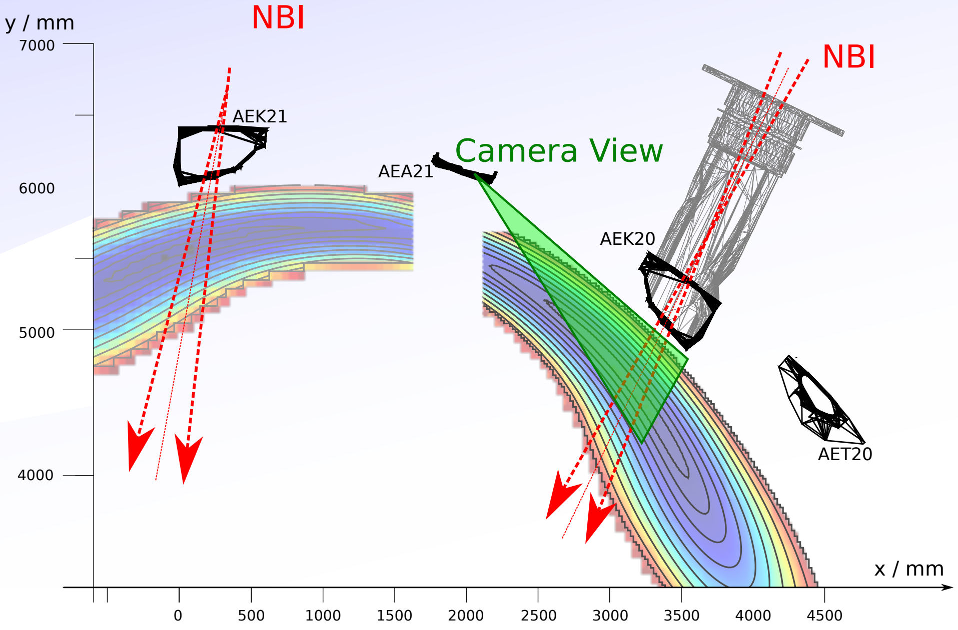

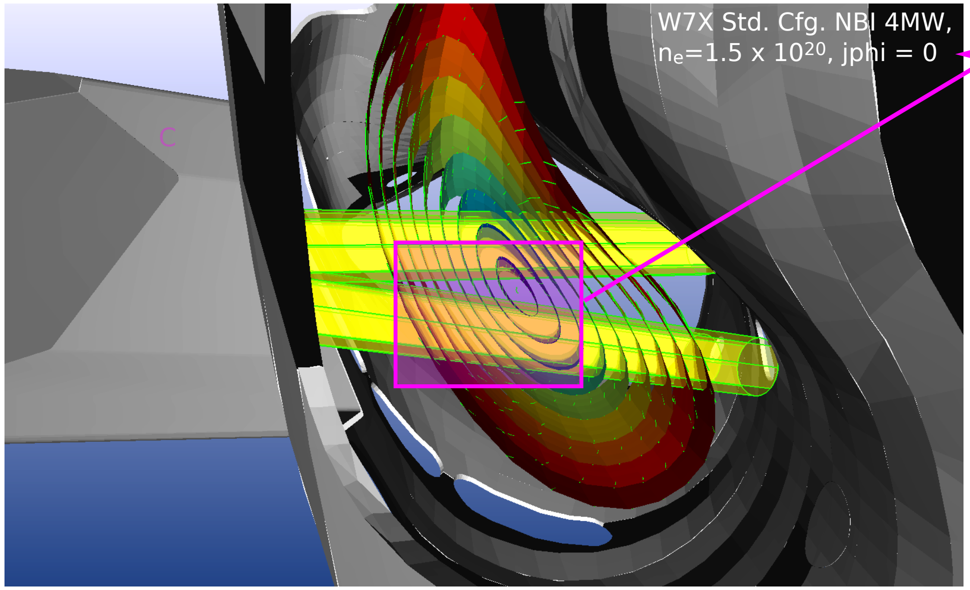

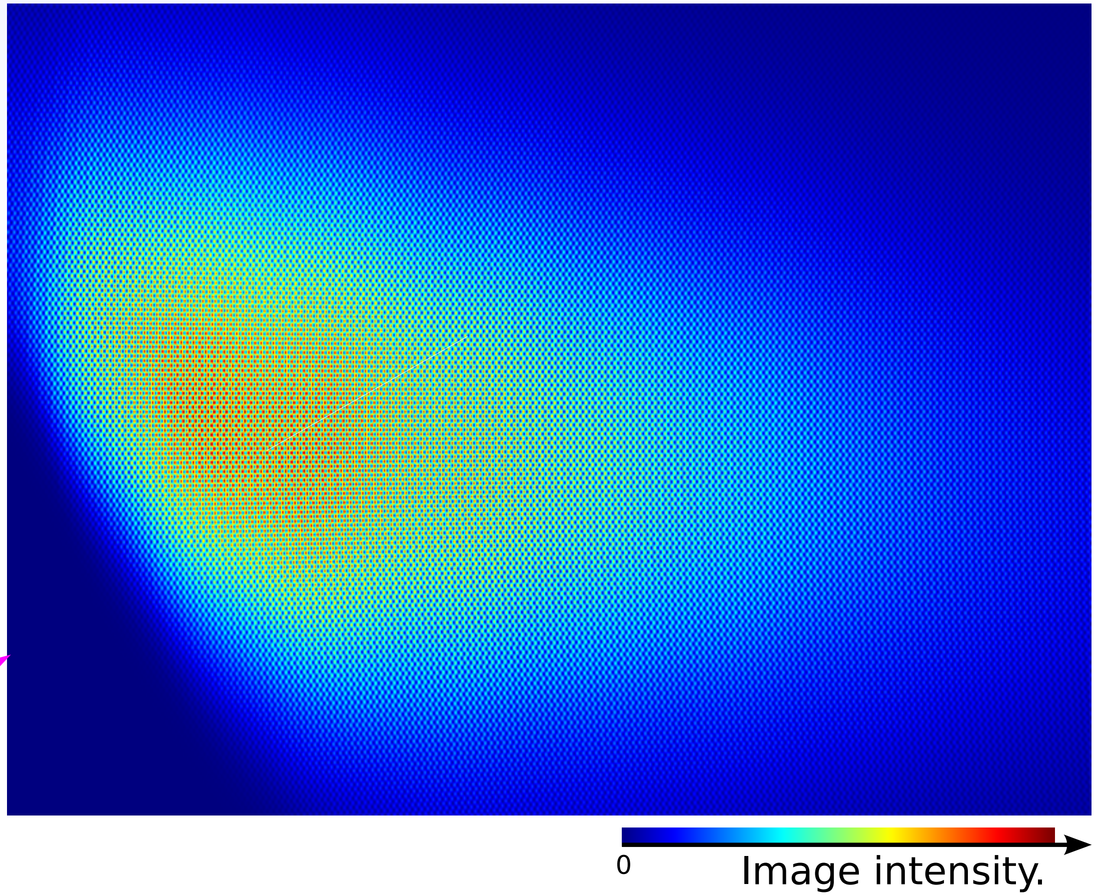

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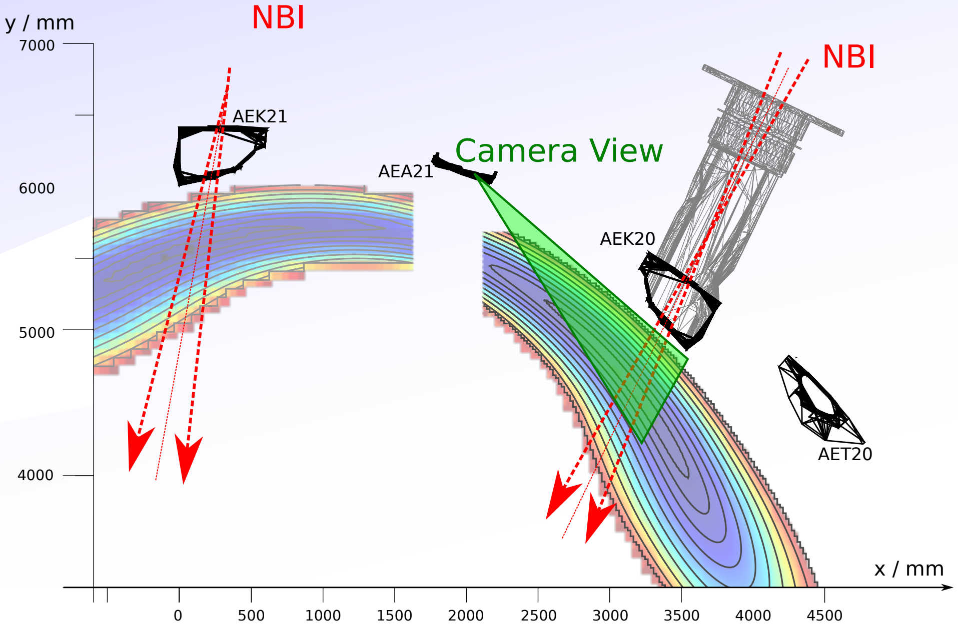

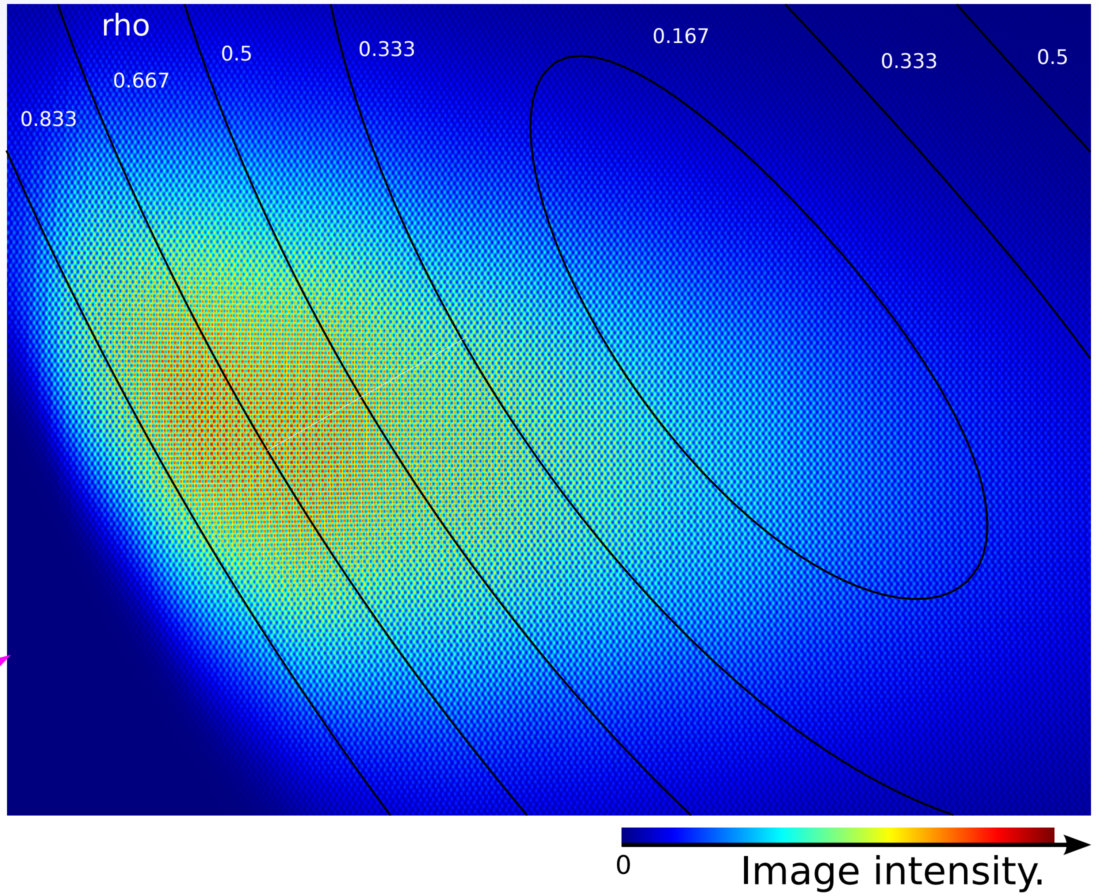

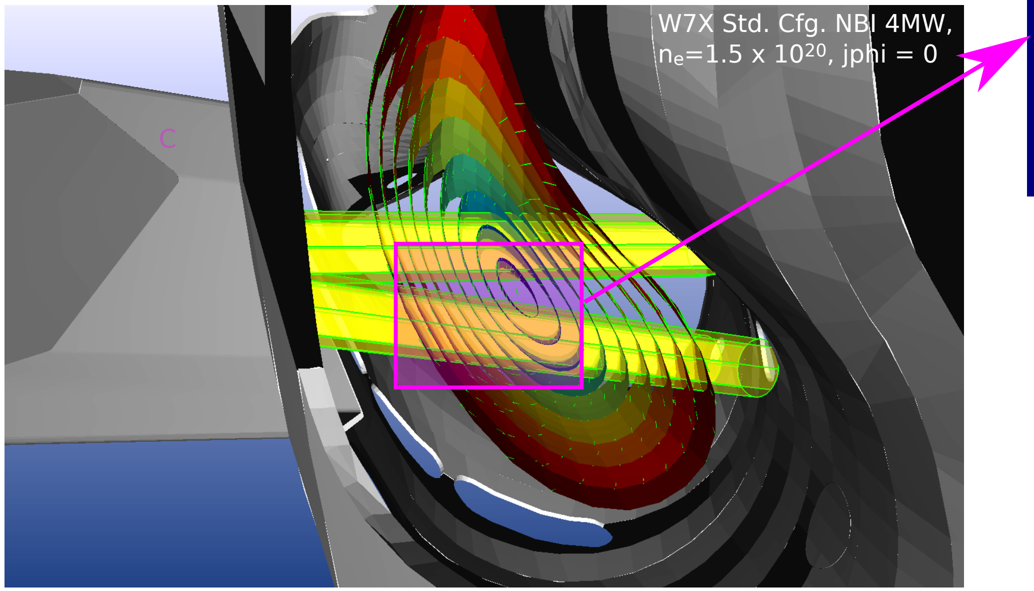

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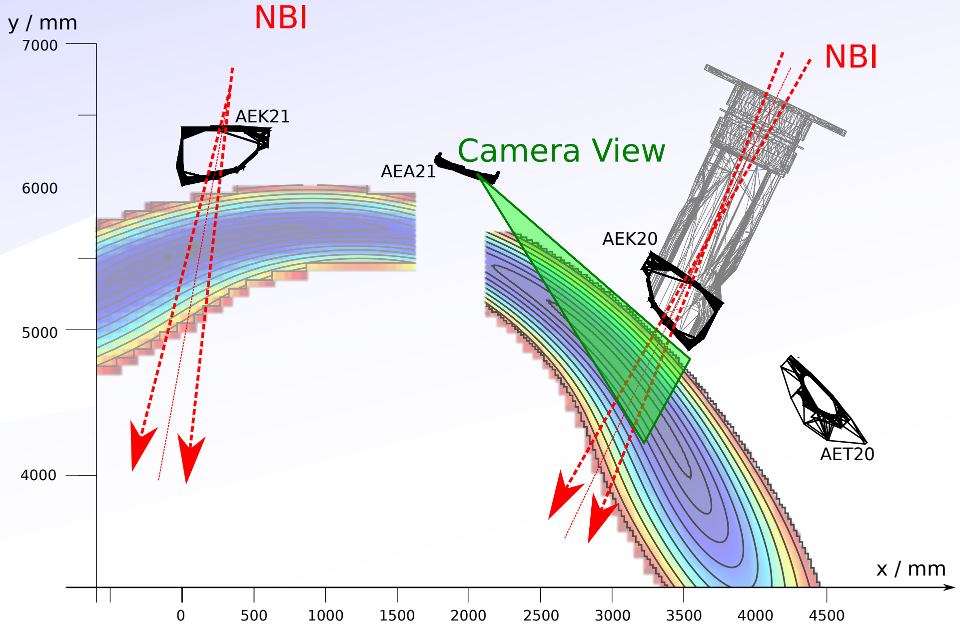

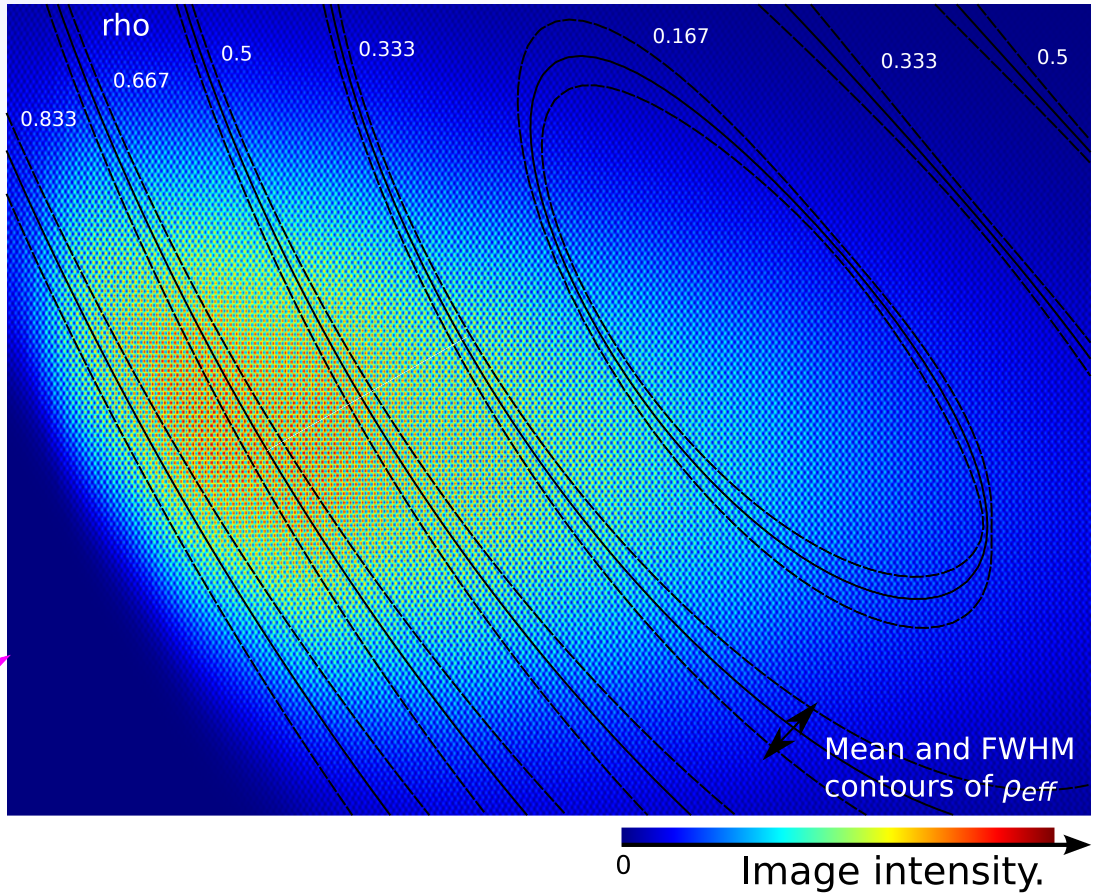

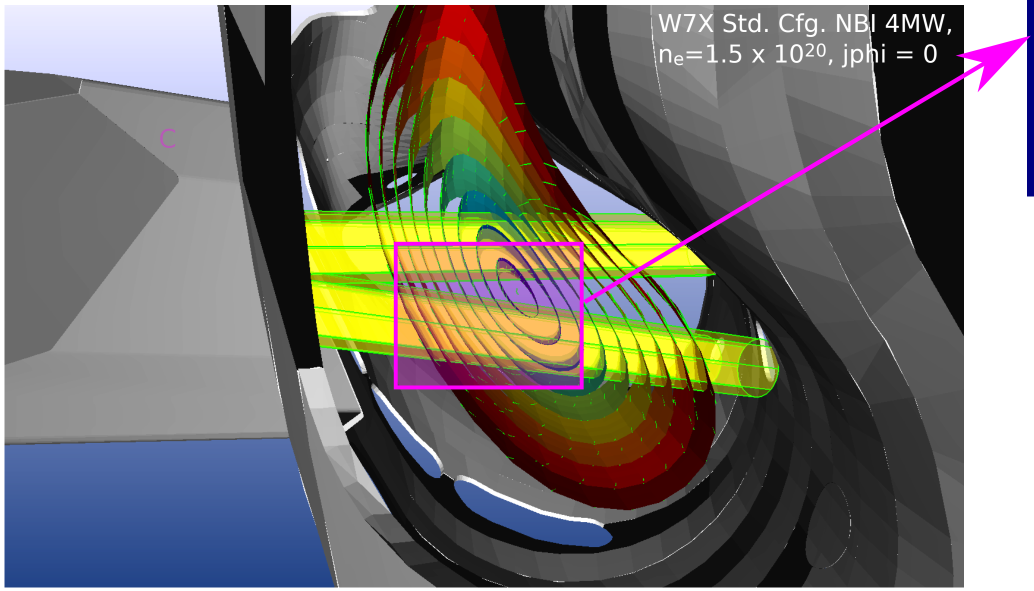

LOS is almost parallel to surfaces
--> Good flux surface resolution.

Geometry (W7X)

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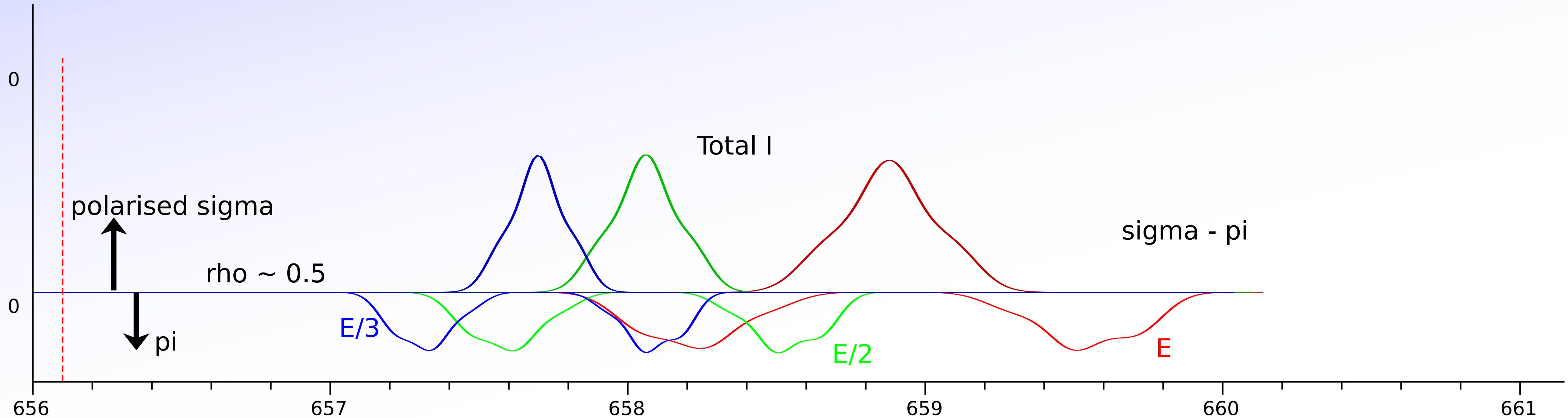

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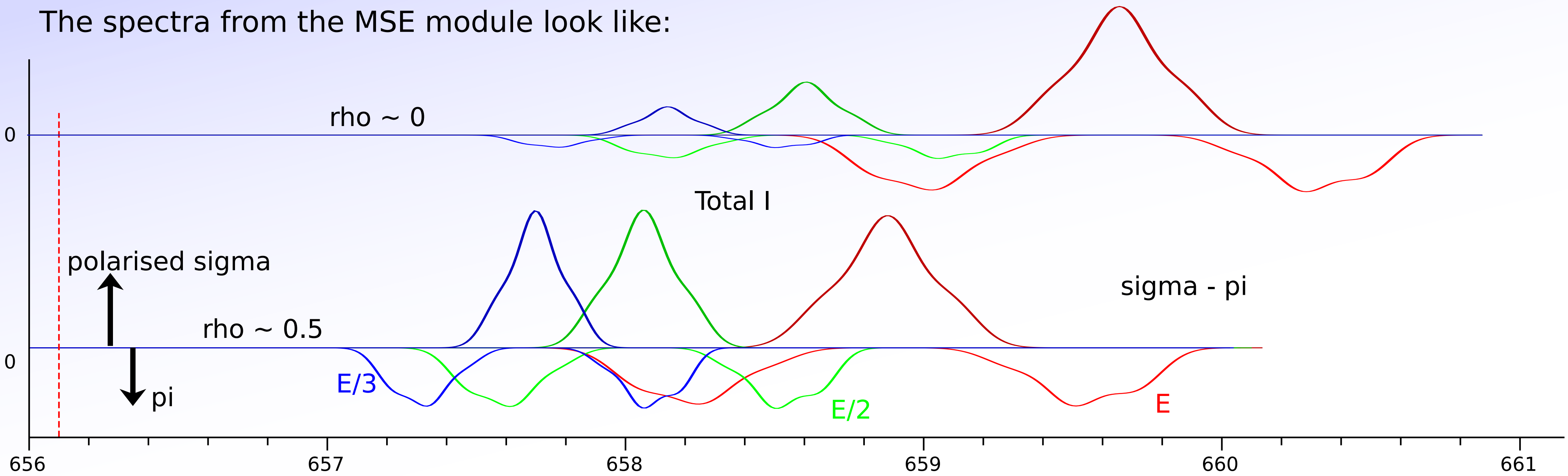

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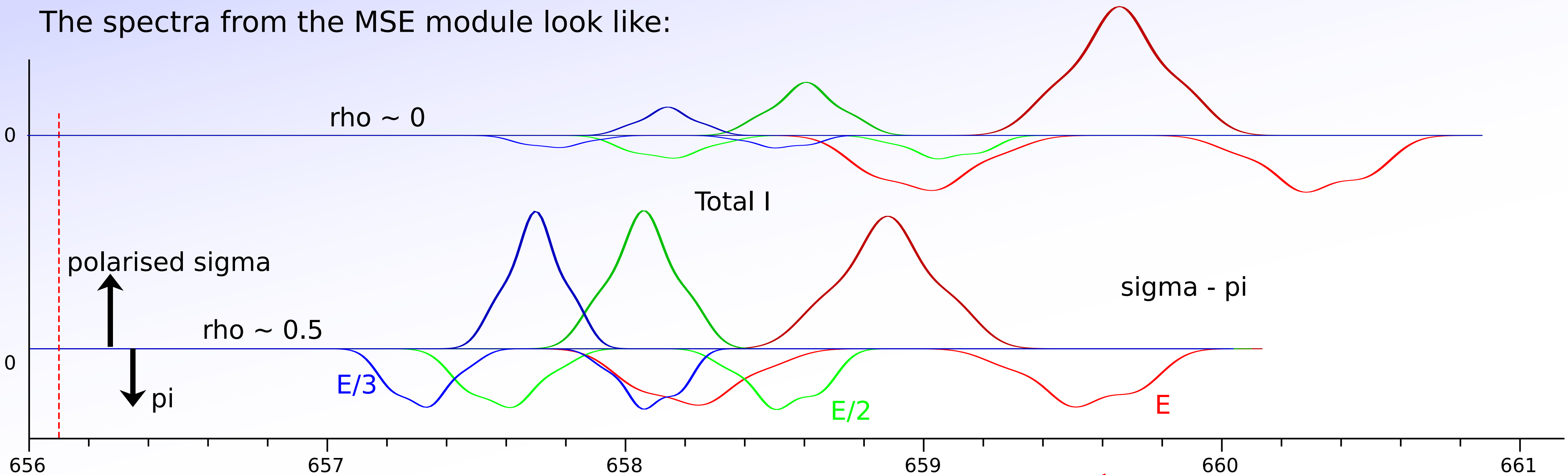

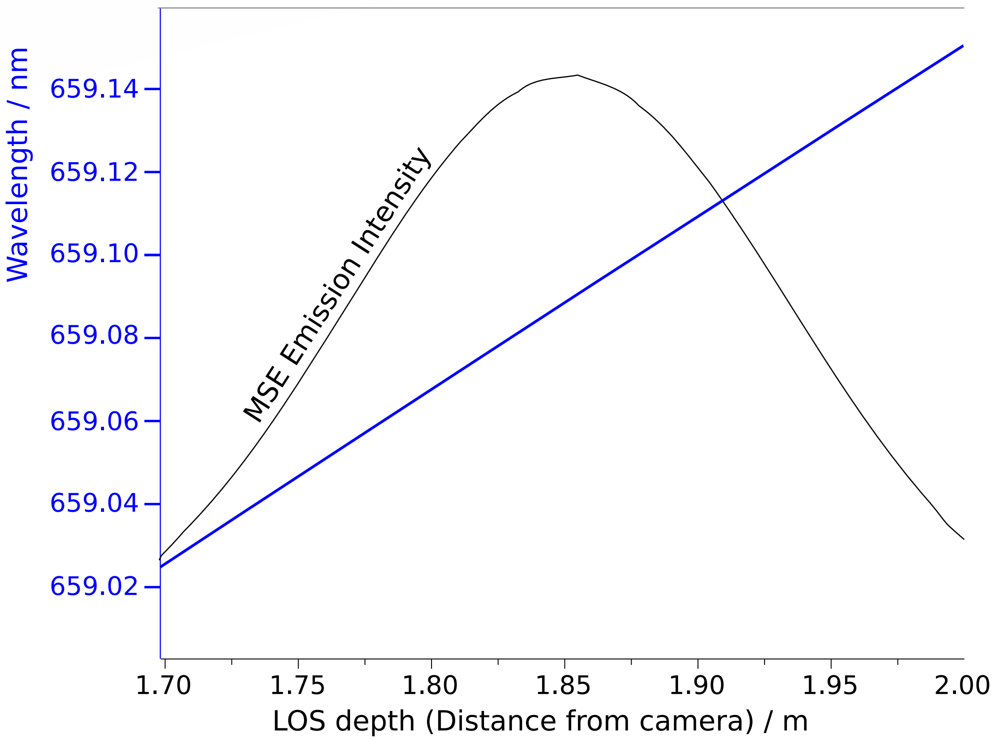

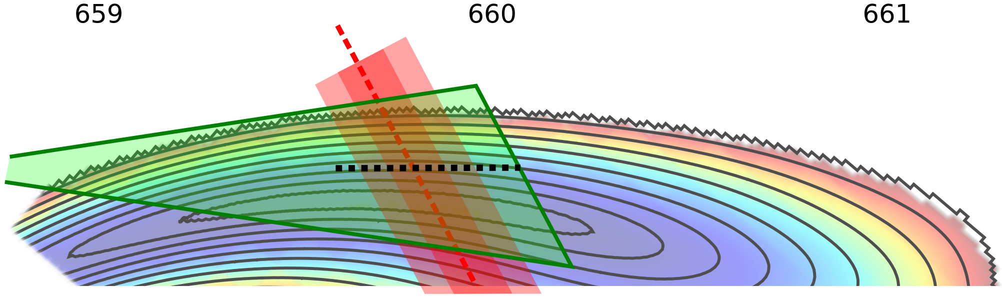

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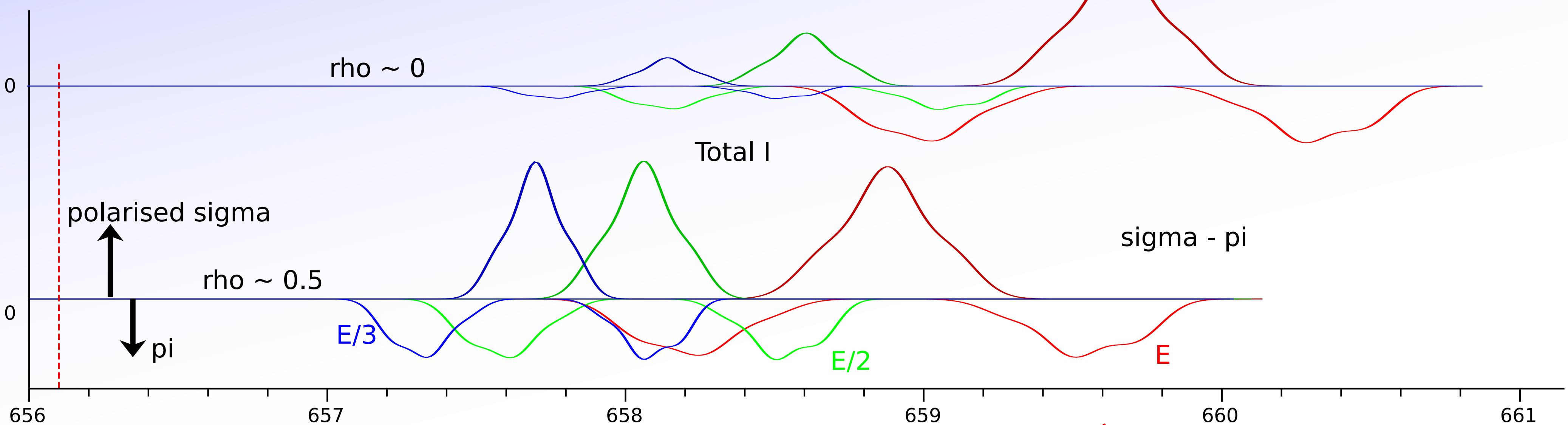

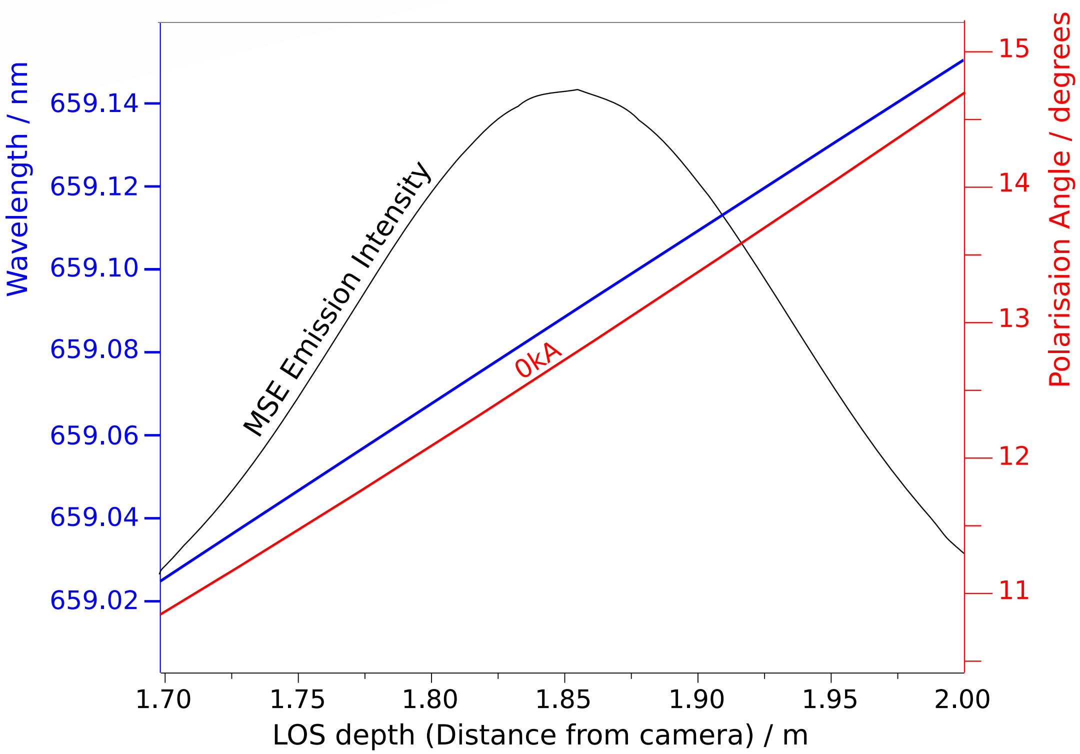

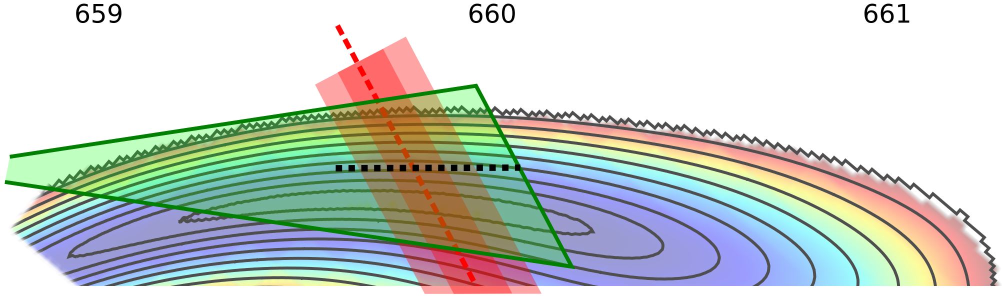

For the Tokamak, pitch change is mostly due to cross-surface LOS integration ~0.6°.

For W7X, pitch changes along ϕ by **>3°.**

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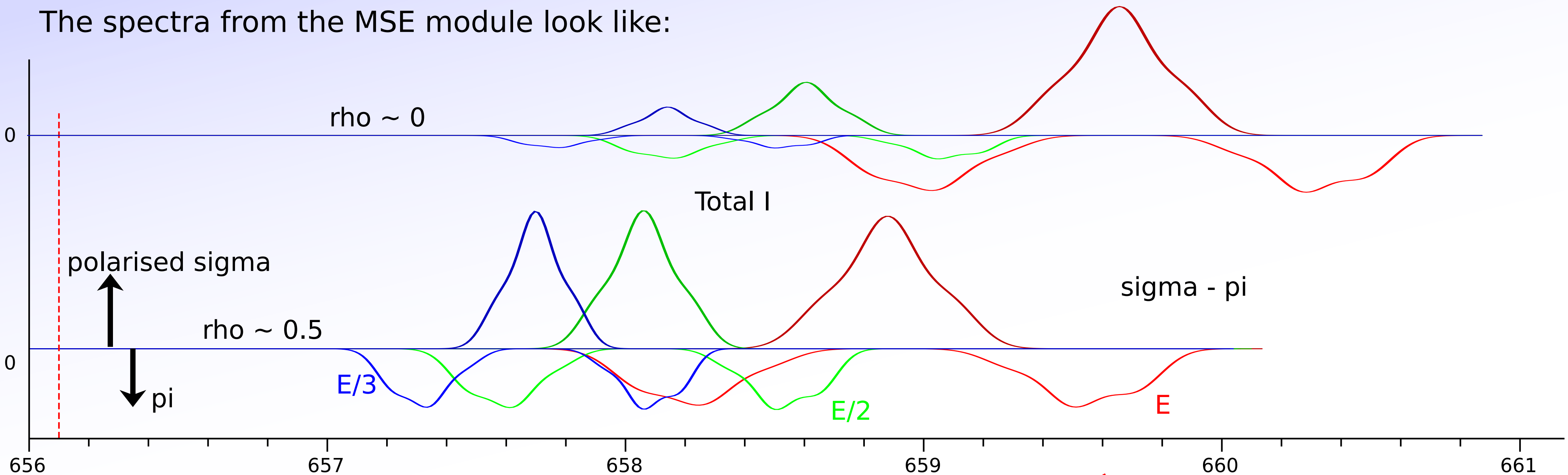

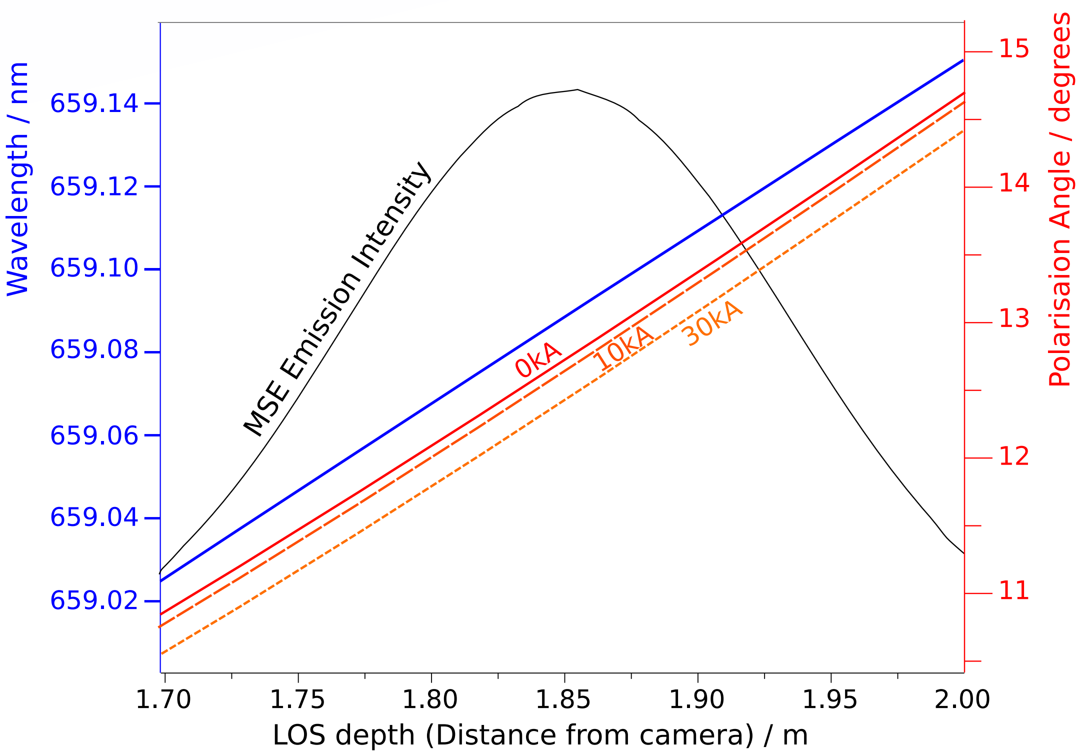

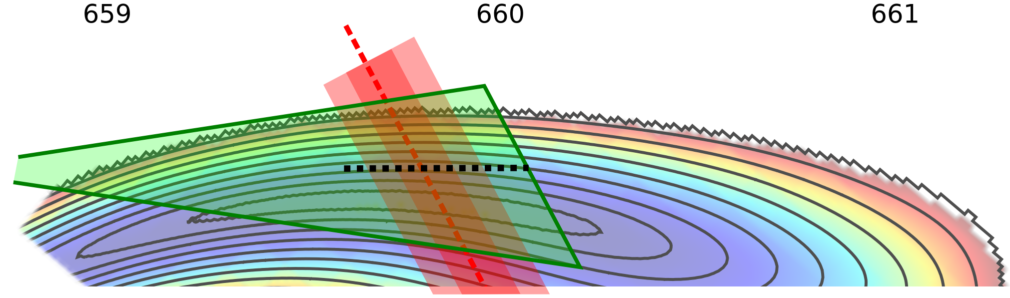

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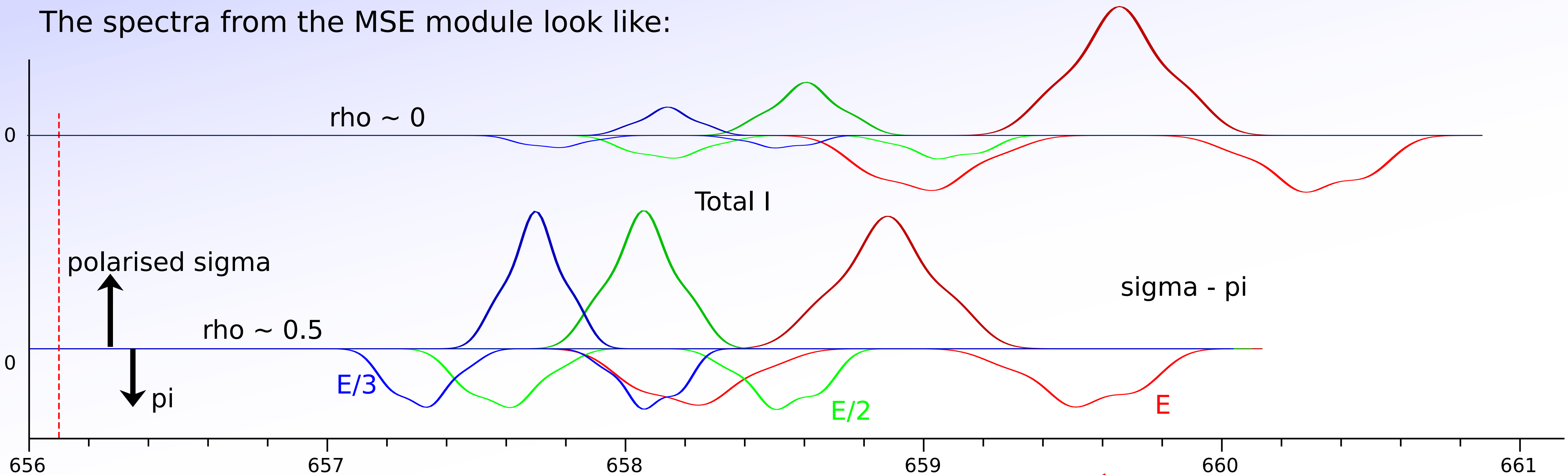

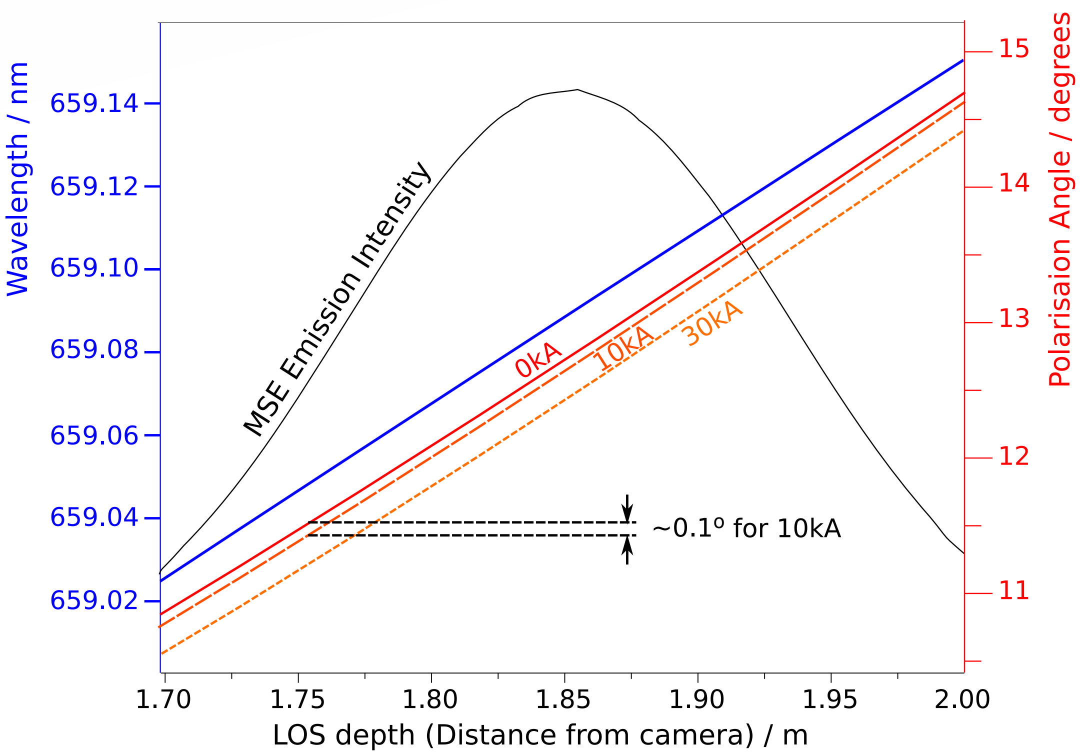

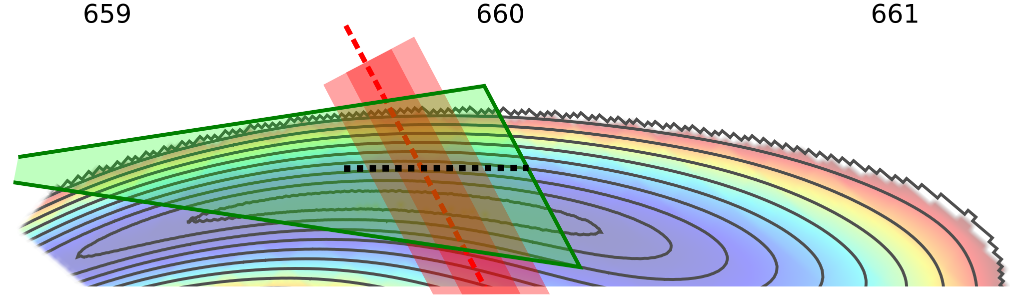

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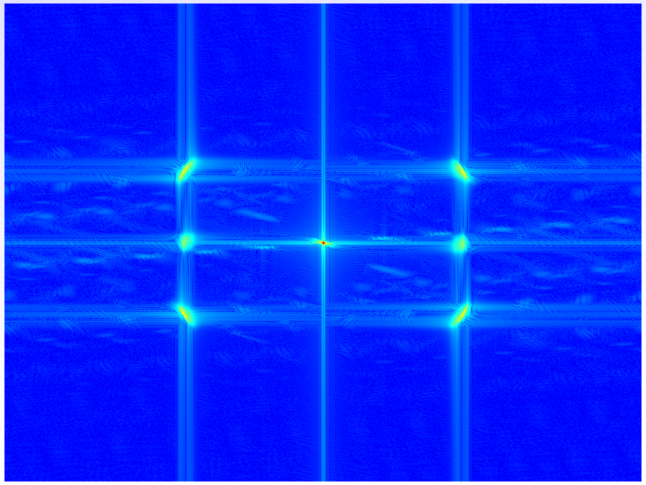
For W7X, pitch changes along ϕ by **>3°.**

A toroidal current of 10kA will give only ~0.1° change, but is constant.

Need to be able to recover the *average* θ.

Demodulation (W7X)

Generated full forward modelled image, add 1% random noise and demodulate:



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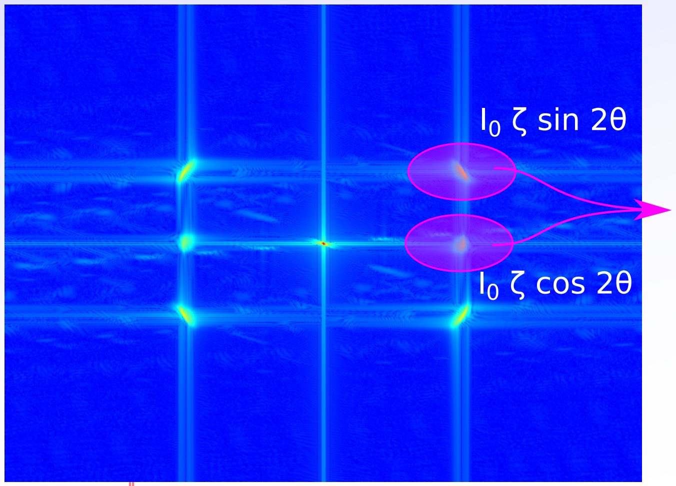

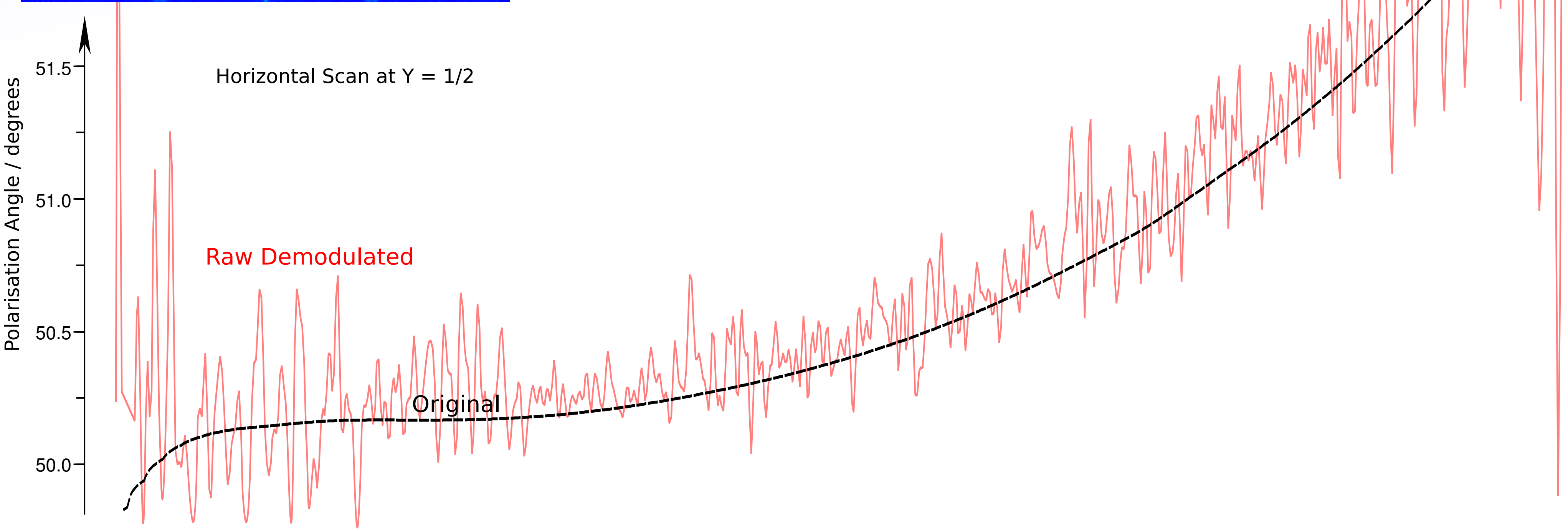

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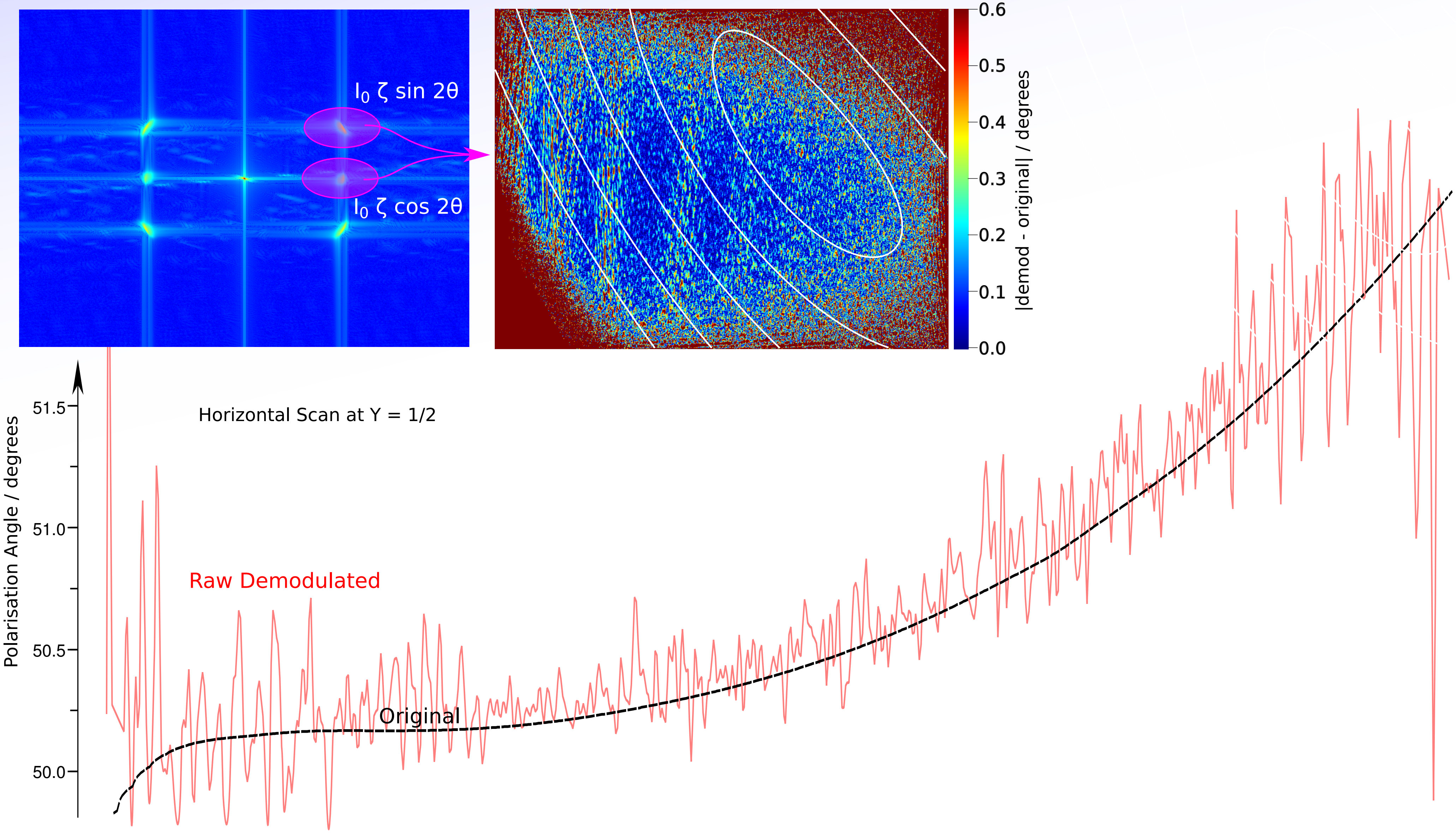

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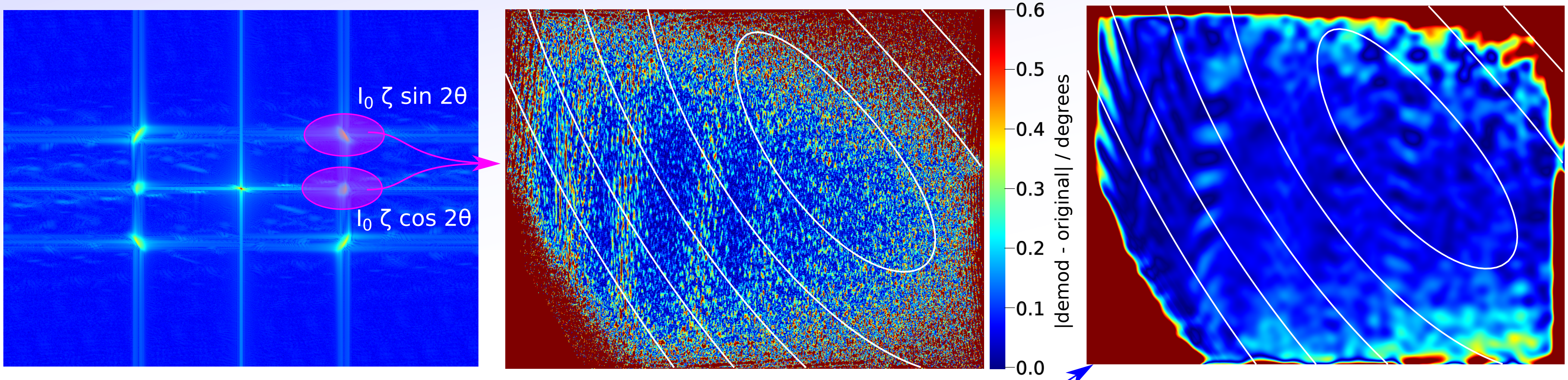
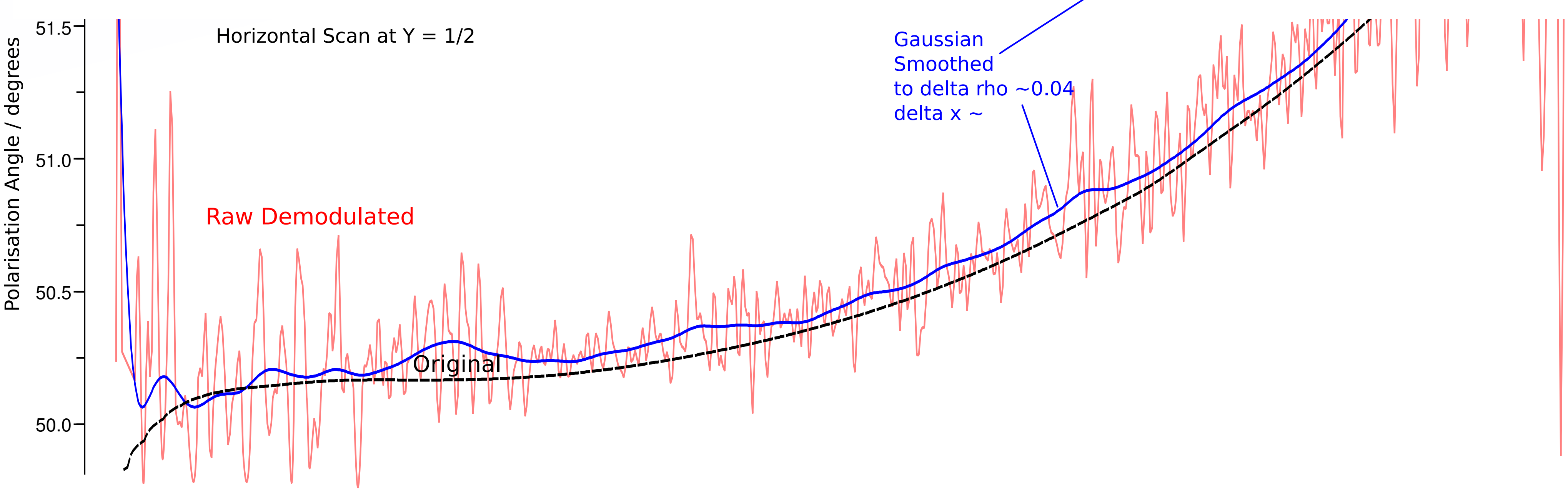
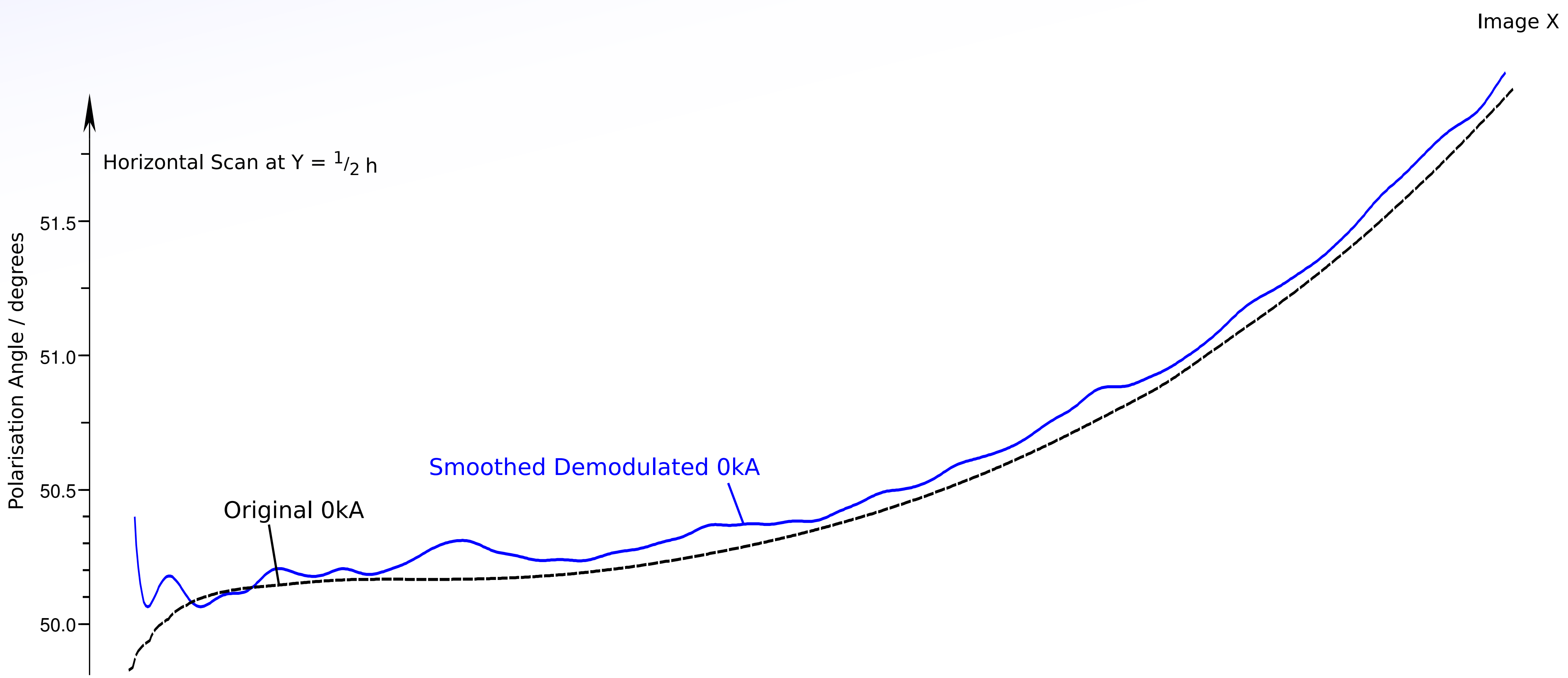




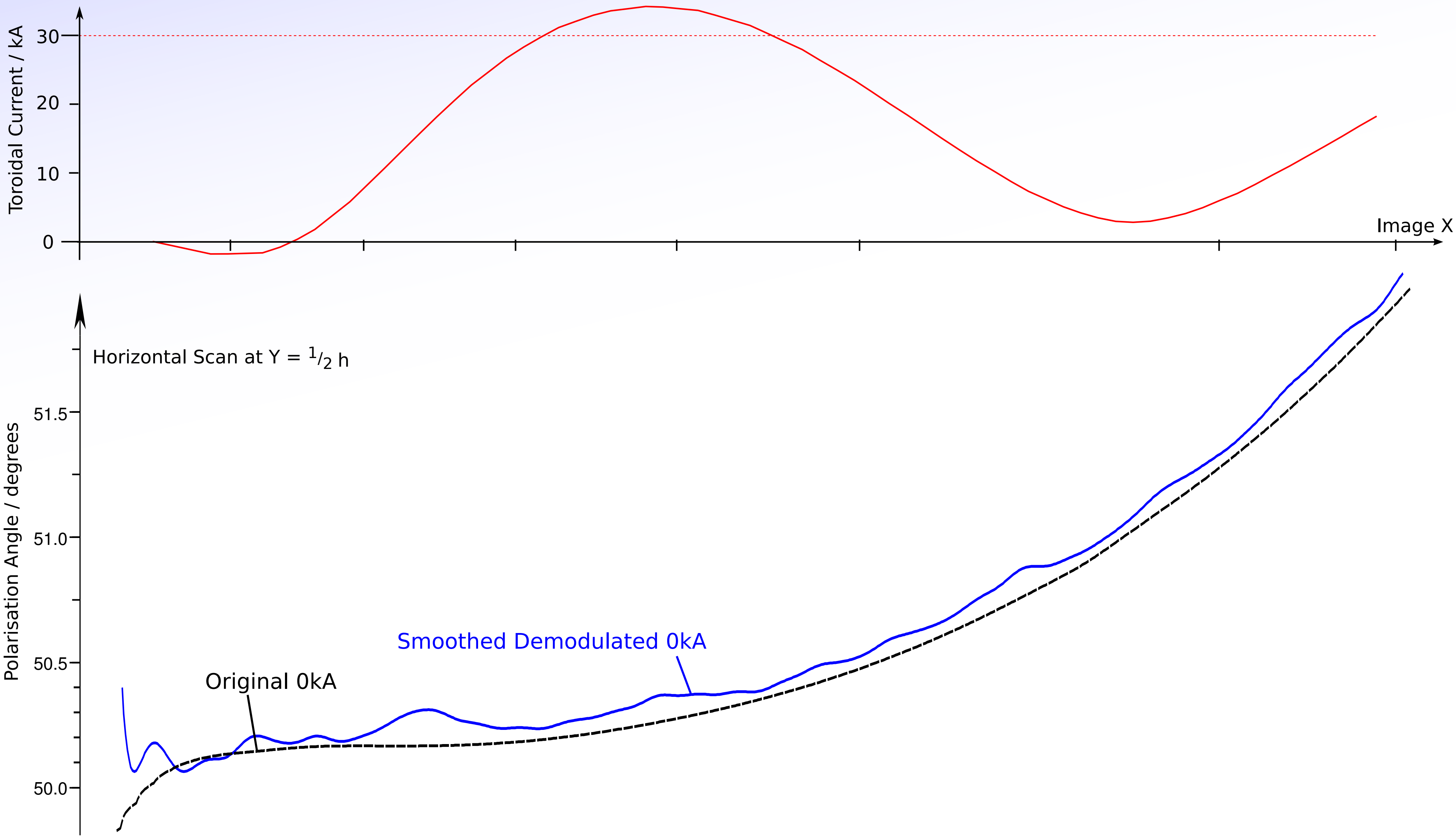
Demodulation not as good as AUG - possibly due to integration of wide range of θ with varying wavelength. Generated image is integration of images but demodulated assuming a single image.

Demodulation Response to jphi (W7X)



Demodulation Response to jphi (W7X)

Add 30kA peak modification to toroidal current profile:



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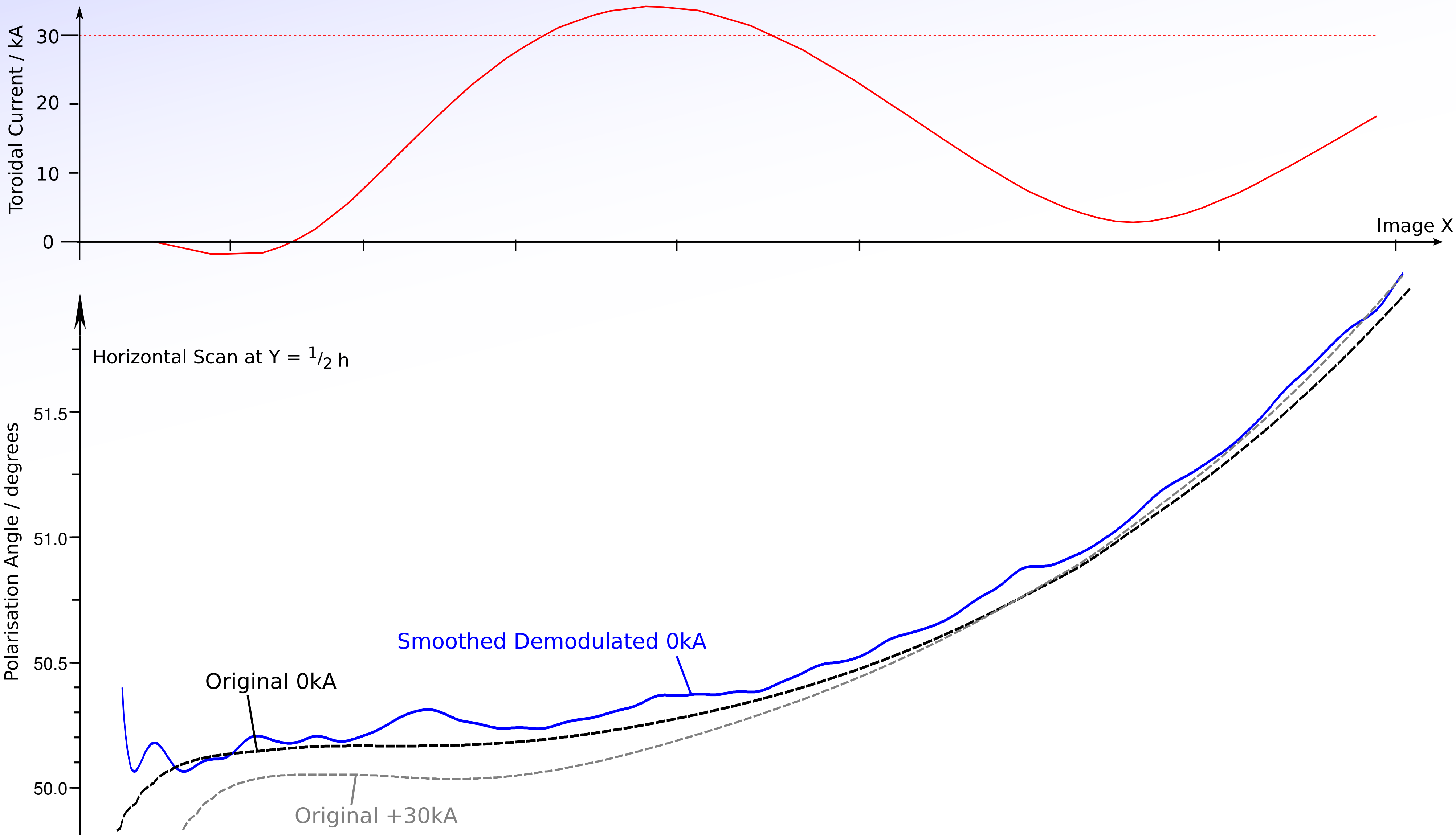

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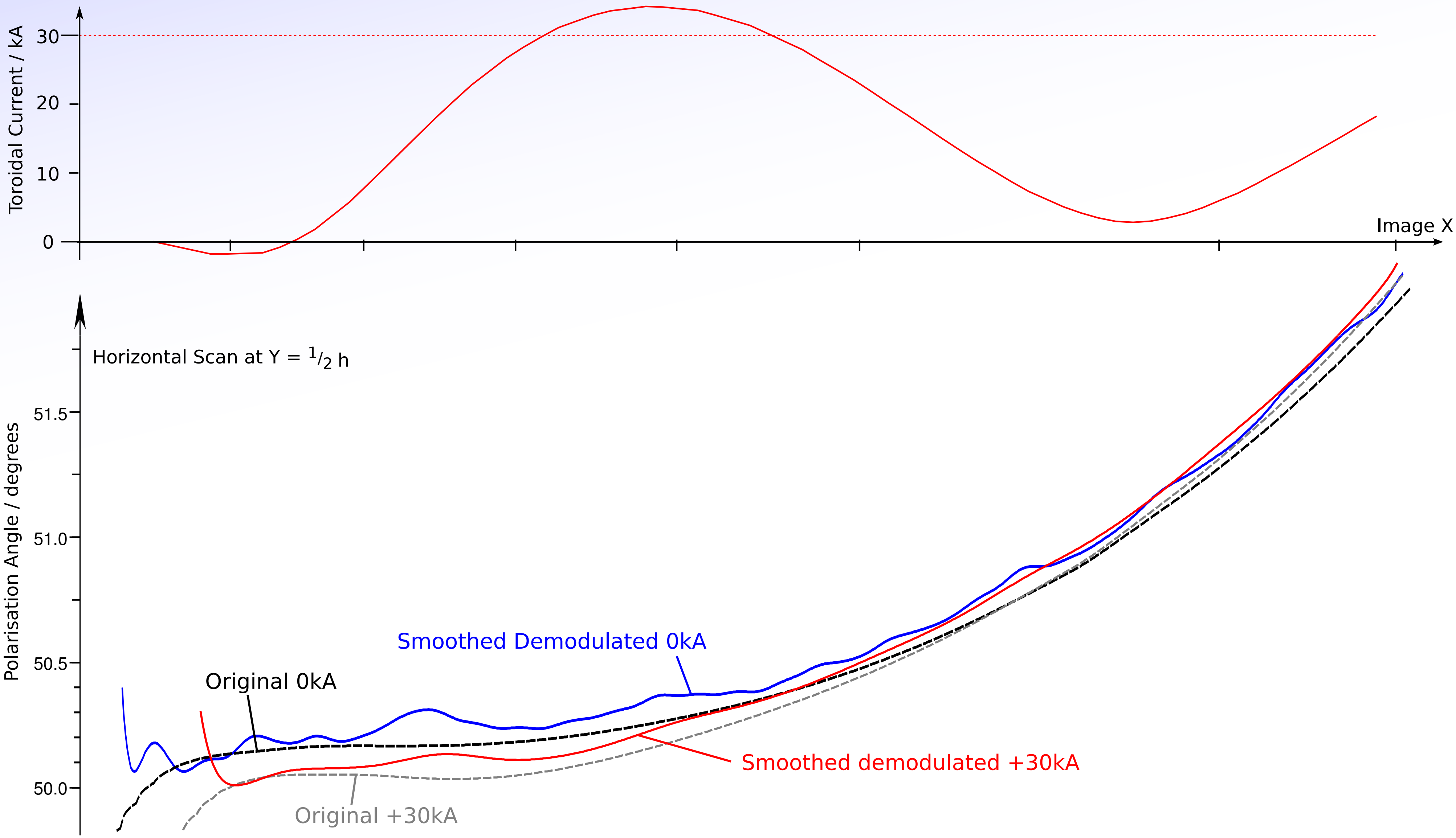

Inference of j_ϕ (W7X)

Can't do the current tomography in 3D (at present).
For a rough idea of the inference capability - use Function Parameterisation in forward model.
Assume fixed/known pressure and coil currents and invert 40x30 polarisation angle map to j_ϕ,
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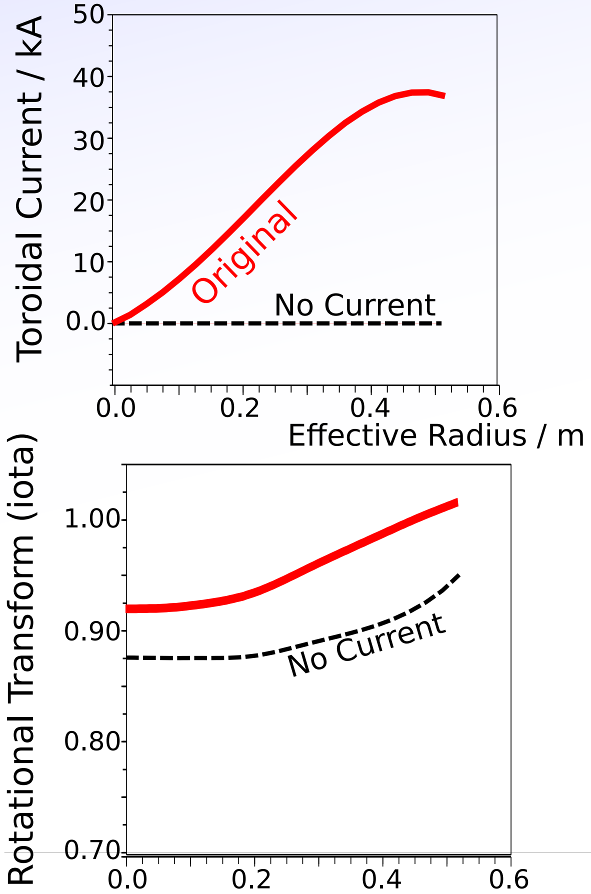

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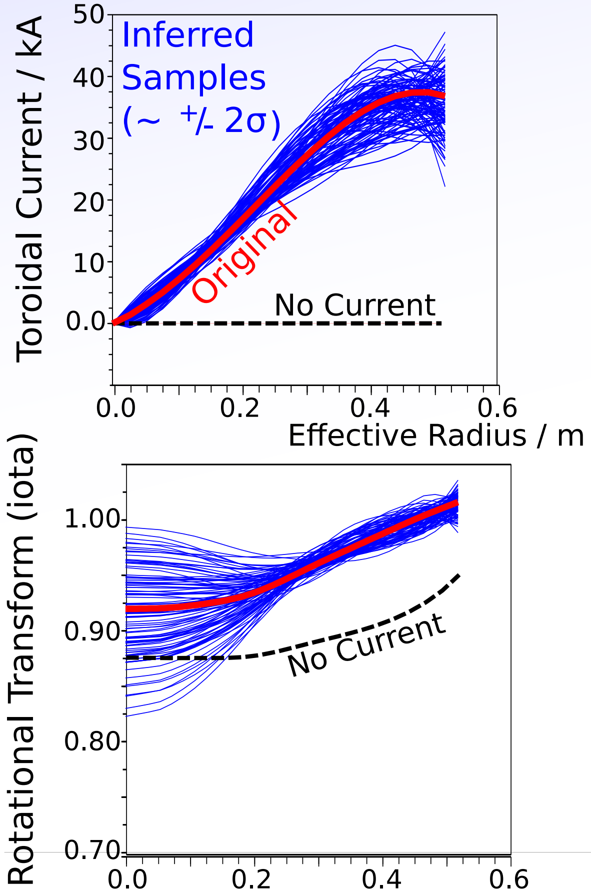

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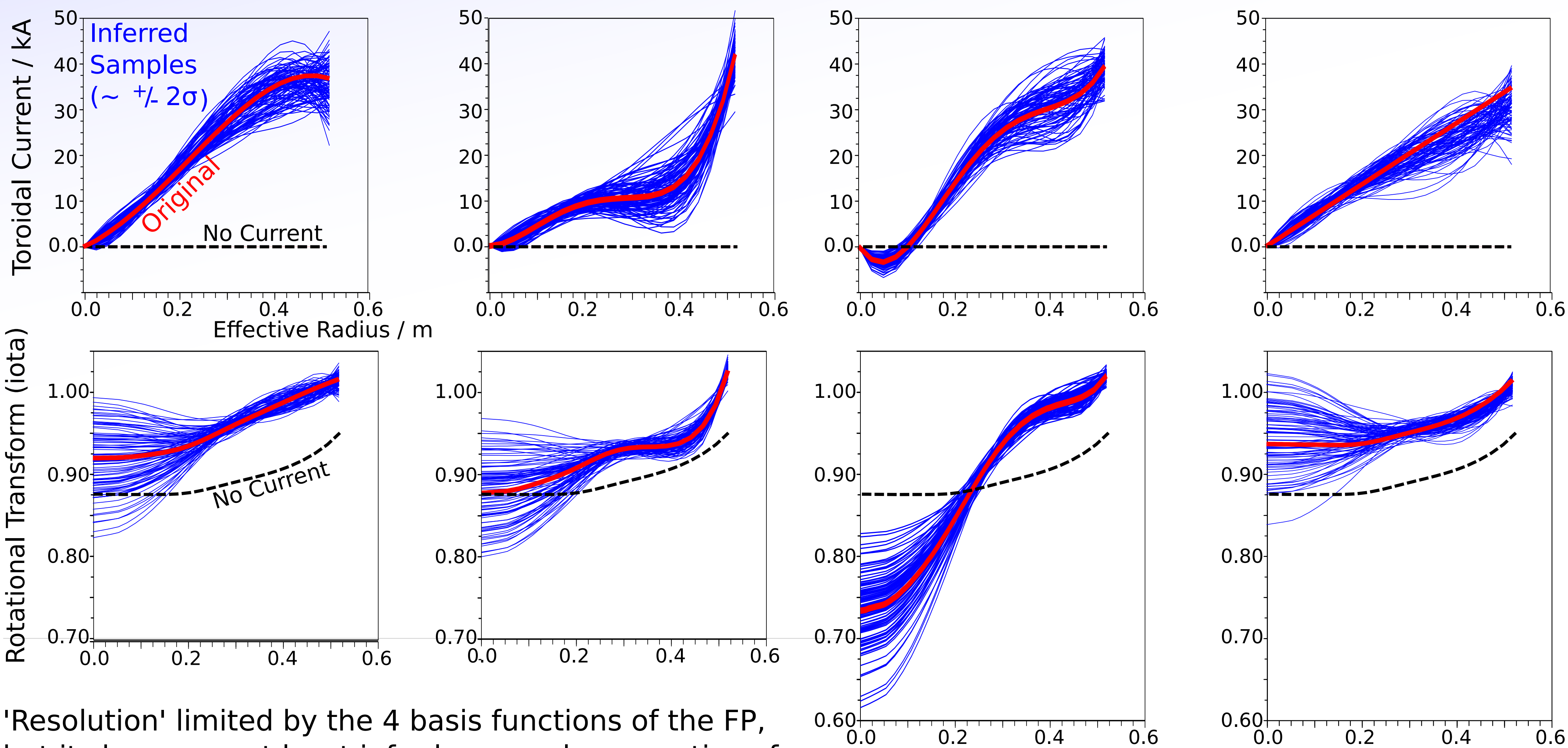


'Resolution' limited by the 4 basis functions of the FP, but it shows can at least infer large scale properties of the bootstrap current profile.
Including other magnetic diagnostics should make it even better.

Summary

- Developed detailed modular forward models for AUG + W7X magnetics, Neutral Beams, MSE and IMSE camera systems.

- Full modelling of the IMSE system under development for ASDEX Upgrade and its capability to infer pitch angle.

- Assesment of ability to directly calculate current in Axisymmetric systems from 2D IMSE measurements.

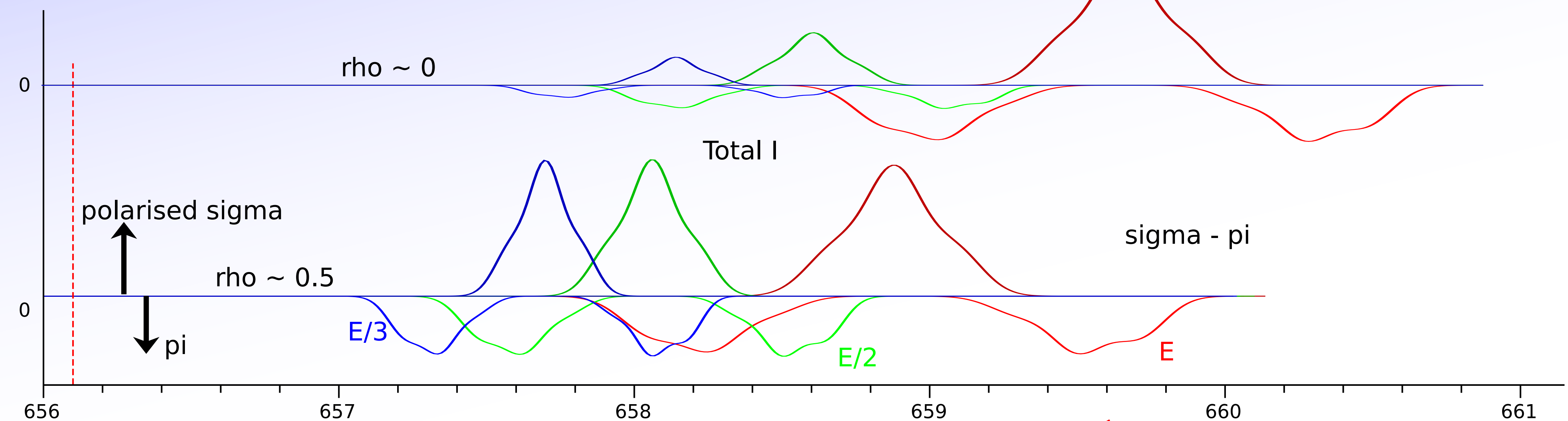
- Simulated Bayesian inference of plasma current from IMSE in ASDEX, without the assumptions associated with equilibrium codes, shows a significant improvement over equivalent 1D measurements.
  - It is relatively easy to also include equilibrium (axisym, isotropic, flow-free) within the Bayesian analysis.

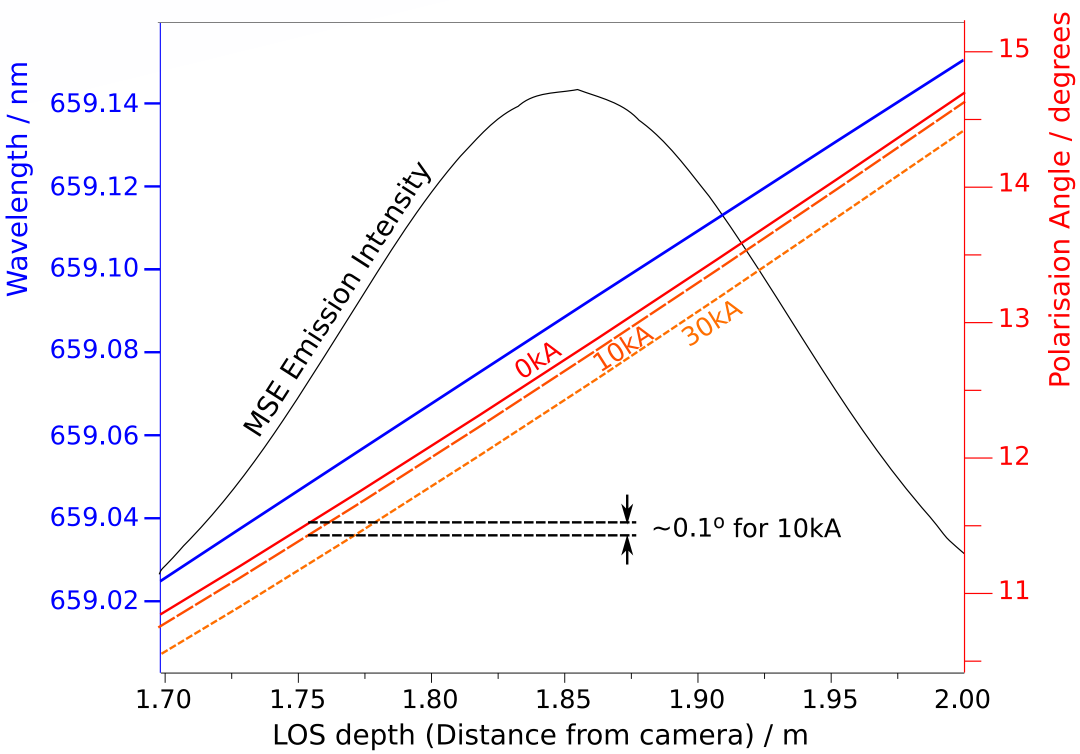
- Modelling of IMSE system for W7X and initial assesment of local average polarisation/pitch angle measurement.

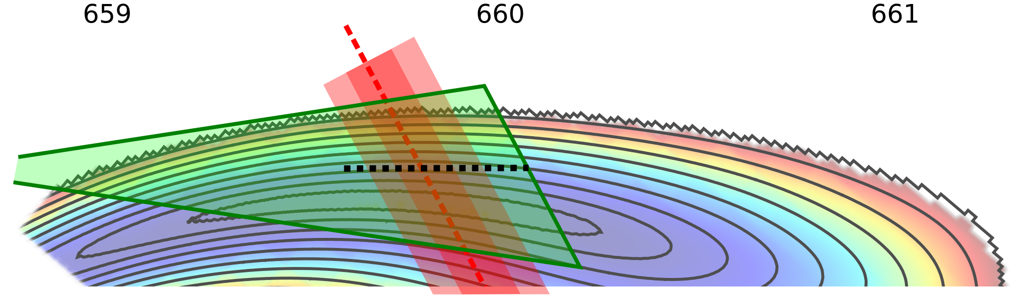
- Assesment of inference of broad information about induced plasma currents in W7X.

Geometry (W7X)

The spectra from the MSE module look like:







For the Tokamak, pitch change is mostly due to cross-surface LOS integration ~0.6°.

For W7X, pitch changes along ϕ by **>3°.**

A toroidal current of 10kA will give only ~0.08° change, but is constant.

Need to be able to recover the *average* θ.